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Identifying the Most Effective Orientation of the CFRP Laminate for Strengthening the Tension Face of the RC T- Beams

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Authors' contributions

This work was carried out in collaboration between all authors. Author MMR designed the study, performed the test and statistical analysis, wrote the protocol, and wrote the first draft of the manuscript. Author MZJ managed the literature searches and made necessary corrections. All authors read and approved the final manuscript.

Research Article

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ABSTRACT

Most of the researches on strengthening so far had been focused on rectangular reinforced concrete (RC) beams. Researches on strengthening of RC T-beams are rather limited. This study focuses on the application of carbon fibre reinforced polymer (CFRP) laminate for strengthening the tension zone of RC T-beam constrained by the presence of a stump (representative of a column) and the effect of varying the length of the strengthening laminates. Three different orientations of the CFRP laminates were tested to evaluate the best orientation. The following orientations were chosen. Orientation 1 was the full application of CFRP laminate along the centre of beam length assuming no stump was present. Orientation 2 was the full application of CFRP laminate alongside the stump parallel to beam length and Orientation 3 was the application of CFRP laminate around the stump and a continuous strip from the side of the stump to the ends of the beam. The beams were tested using the three point bending test set-up. The most suitable orientation of CFRP laminate determined was Orientation 2. The load carrying capacity had increased by about 70% compared to un-strengthened beam by strengthening both the tension and compression zone.

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Keywords: Orientations of CFRP; strengthening; continuous beam; carbon fiber; structure; negative moment.

1. INTRODUCTION

While many methods of strengthening structures are available, strengthening by applying CFRP laminate has become popular. For strengthening purposes, application of CFRP laminate is more advantageous than other materials. Teng et al. pointed out that, there is increased demand for extensive research work to improve the characteristic behaviour of FRP materials to establish their application acceptability in RC structures, beams, slabs and columns [1]. In particular, their practical implementations for strengthening civil structures are numerous.

Several researchers pointed out that most of the pragmatic works consist mainly of the rectangular beams [2-5]. Furthermore, the design methodologies as well as guidelines are evolved mainly for the simply supported rectangular beams. Generally, the research works were conducted on RC rectangular sections which are not truly representative for the fact that most RC beams would have a T- Section due to the presence of a top slab.

Although many research studies had been conducted on the strengthening and repairing of simply supported RC beams using external plates, there is little reported work on the behavior of strengthened RC-T beams [6-9]. Especially, works relating to the application of CFRP laminate for strengthening the tension zone of RC T- beams in the presence of column are very few. In addition, there are few difficulties arise due to the presence of columns and other components such as electric and plumbing lines or HVAC ducts. These columns and components hinder the process of applying CFRP laminate in this region using conventional techniques. Another important point is that, the use of thick steel plates for strengthening will raise the floor level, which might be undesirable.

An exhaustive literature review has revealed that, a little amount of research works had been done to address the possibility of strengthening the tension zone of RC T- beam in presence of column using FRP materials .The constraints caused by columns in the application of the strengthening system were not considered in the existing researches.

2. PREVIOUS RESEARCH WORKS RELATED TO THIS TOPIC

Jumaat et al. pointed out that, although several research studies have been conducted on the strengthening and repair of simply supported reinforced concrete beams using external plates, there are few reported works on the behavior of strengthened T-beams in the presence of column [10]. Furthermore, almost all the available design instructions to strengthen the structures by the external laminates of FRP are demonstrating the simply supported beams [11-13]. The literature review revealed that a meager amount of research works had been explored to address the potential of applying CFRP laminate for strengthening the tension zone of RC ‘T’– beam in the presence of column .

On the field of strengthening continuous beam, Grace et al. tested five continuous beams [14]. They found that the use of FRP laminates to strengthen continuous beams is effective for reducing deflections and for increasing their load carrying capacity. They also concluded that beams strengthened with FRP laminates exhibit smaller and better distributed cracks. Later, Grace et al. investigated the experimental performance of CFRP strips used for flexural strengthening in the hogging region of a full-scale reinforced concrete beam [15]. Grace et al. also worked on another research where three continuous beams were tested [16]. They noted that CFRP strips were not stressed to their maximum capacity when the beams failed, which led to ductile failures in all the beams. On the other hand, El-Refaie et al. (2003a) examined eleven reinforced concrete (RC) two-span beams strengthened in flexure with external bonded CFRP sheets [17]. In another research, El-Refaie et al. tested five reinforced concrete continuous beams strengthened in flexure with external CFRP laminates [18]. They investigated that extending the CFRP sheet length to cover the entire hogging or sagging zones did not prevent peeling failure of the CFRP sheets**.** They also found that, strengthened beams at both sagging and hogging zone produced the highest load capacity**.** Ashour et al. tested 16 reinforced concrete (RC) continuous beams with different arrangements of internal steel bars and external CFRP laminates. As in previous studies, they observed that increasing the CFRP sheet length in order to cover the entire negative or positive moment zones did not prevent peeling failure of the CFRP laminates[19]. Aiello et al. compared the behavior between continuous RC beams strengthened with of CFRP sheets at hogging or sagging regions and RC beams strengthened at both sagging and hogging regions [20]. Recently, Maghsoudi and Bengar (2008) have examined the flexural behavior and moment redistribution of reinforced high strength concrete (RHSC) continuous beams strengthened with CFRP [21]. Finally, Akbarzadeh and Maghsoudi (2010) have conducted an experimental program to study the flexural behavior and moment redistribution of reinforced high strength concrete (RHSC) continuous beams strengthened with CFRP and GFRP sheets [22].

In all the above cases it is seen that the researches were conducted on RC rectangular sections which are not representative of the fact that most RC beams would have a T-Section due to the presence of top slab. In all the above cases, the restraint caused by the columns in the application of the strengthening system was not considered.

3. EXPERIMENTAL PROGRAM

A total of four; 3300mm long, 325mm deep, 380mm x 100mm flange, T-shaped RC beams are fabricated for this experimental endeavor. Beams B0, B1, B2 and B3 are selected for selecting the best orientation. The orientations are shown in Figs. (5-7). The stump of height 150mm and cross section 150 x 150 mm was cast in the middle of the flange to represent the intersection of the beam with a column. The objective of casting this stump is to provide restraints in the application of the strengthening system at the mid-section of the beam. In actual field situation, the flange portion of RC-T beam in the beam column intersection remains in tension. The test setup is arranged in such a way that the flanges of T beams are in tension to represent the actual field condition. It is done by applying the load on inverted T beam. Three point bending is applied, where the supports represents the points of inflection of a continuous beam where the bending moment is zero. The detailed test matrix is shown in Table 1.

Table 1. Test matrix

Beam name	**Concrete strength (MPa)**	**CFRP laminate**			
		Size (mm)	**Length**	**Applying zone**	**CFRP orientation**
B 0	37	not applicable			
B 1	39	100 × 1.4	3000 mm	Tension zone	Orientation 1
B 2	40	100 × 1.4	3000 mm	Tension zone	Orientation 2
B 3	42	100 × 1.4	All four sides of stump	Tension zone	Orientation 3

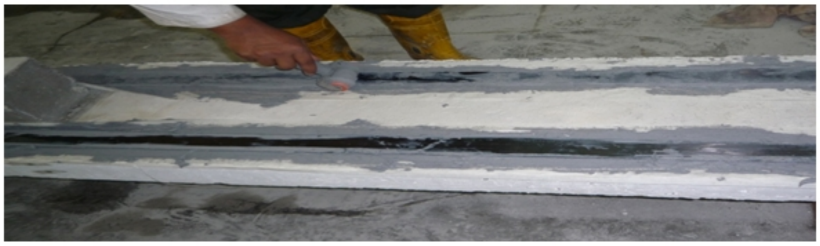
After 28 days of curing, the beams are strengthened with CFRP laminate. 1.4 mm x 100 mm CFRP laminate (SikaCarboDur S1014/180) has been used for all the strengthened beams. Oil, dirt and other foreign particles removed from the surface in order to expose the texture of aggregate with the help of a diamond cutter. The dust particles were removed by high pressure air jet (Fig. 1). Colma cleaner is used to remove the carbon dust from the bonding face of CFRP laminate.

Fig. 1. Preparation of surface

CFRP laminate was bonded to concrete surface by using Sikadur-30 as a bonding agent. The process of mixing the adhesive and applying it to the surfaces are shown in Fig. 2. The well–mixed adhesive is pasted to the bonding surface of concrete up to 2-3 mm thickness.

Ready mixed concrete has been used for this research. Concrete compression testing has been carried out at 7th day, 28th day and the day of testing. Three cubes are tested each time and the average strength is calculated. The average compressive strength is 26 MPa for beams B4, B5 and B7. The flexural tensile strength of concrete is estimated using the equation of, $f = 0.53\sqrt{fc'}(kg/cm^2)$.The average yield strength and ultimate strength were 560 MPa and 645 MPa respectively. Modulus of elasticity of the bar is 200GPa.CFRP laminates of type SikaCarboDur S1014/180; (1.4mm x 100 mm) has been used. The maximum design and ultimate strain of CFRP laminates are 0.85% and 1.7% according to the manufacturer's guideline. The tensile strength is 2800MPa. Modulus of elasticity is 165GPa.

(a) Mixing of adhesive (b) Placing of adhesive on CFRP laminate



(c) Pressing CFRP with the help of Rubber Roller

Fig. 2. Installing CFRP

Strain gages (30 mm) are attached to main reinforcing bars, CFRP laminates and on to the concrete surface to measure their strain. The reinforcing bars are smoothed by grinding machine, CFRP laminates are cleaned with acetone and concrete surface are also smoothed before fixing strain gauges. All strain gauges were connected with data logger to record the strain values during the test. LVDT (50mm capacity) has been used to measure the mid-span deflection of the beam. The LVDT was connected to Data Logger to record the readings during test. Data Logger TDS-530, manufactured by Tokyo Sokki Kenkyujo Co, Ltd. has been employed to record data from various connections. Strains at the sides of the beams were measured from the demec points using digital extensometer. The crack width of the beams during test was measured by using Dino-lite this instrument (Fig. 3).

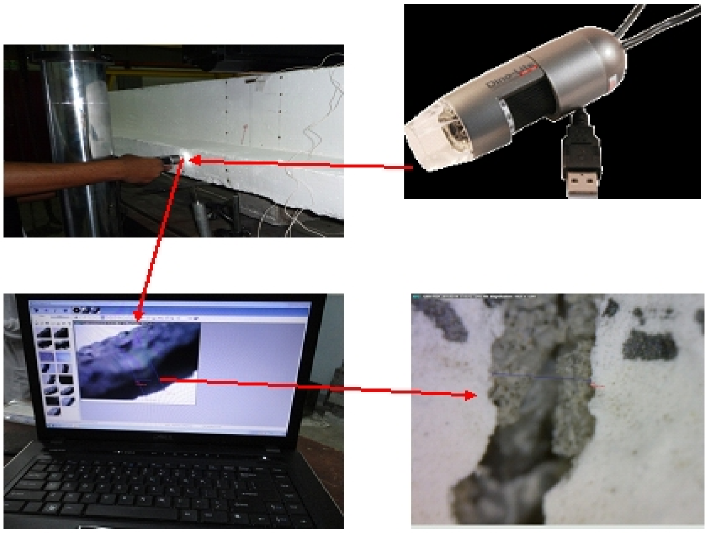


Fig. 3. Measuring crack width by using Dino-lite

Three point bending was applied, where the supports represented the points of inflection of a continuous beam where the bending moment is zero. The position of the load as well as the setting of the machine is shown in Fig. 4. After the beam has been lifted and positioned on the supports, the LVDT was placed and after that all the strain gages as well as the LVDT were connected to data logger properly. The load was applied with the help of INSTRON SATEC Testing Machine. This machine can apply up to 600 kN load and this machine can be controlled automatically by computer. The loading rate was controlled by **Blue Hill software**. The loading rate was controlled by 6 kN /min up to 60 kN. At every 10 kN the loading was hold for 2 minutes so that the manual readings from demec and manual readings of deflection can easily be taken. All other readings were recorded by Data Logger at every 10 second. The cracking width was measured with the help of 'Dino-lite'. The design and analysis is shown in Appendix: A

Fig. 4. Test setup

Fig. 5. CFRP orientation 1 (B1)

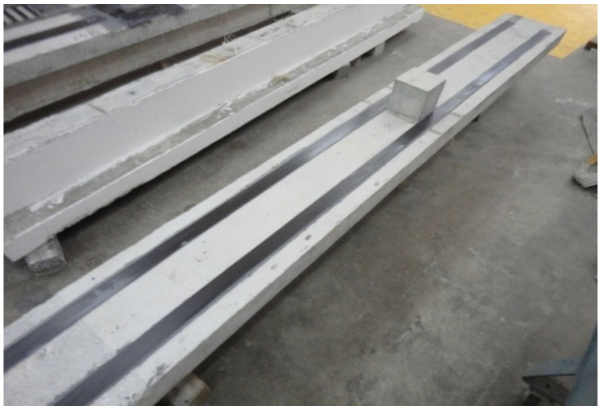


Fig. 6. CFRP orientation 2(B2)

Fig. 7. CFRP orientation 3 (B3)

4. PERFORMANCE OF DIFFERENT ORIENTATION OF CFRP LAMINATE

In this section, four beams are presented. The cross section and length of CFRP of all the strengthened beams in this section are same but the orientations of CFRP laminates are different. The control beam (B0) is without column stump and is not strengthened. Second beam (B1) is also without column stump and strengthened in the tension zone only (Fig. 5). The third beam (B2) is with column stump and the orientation of CFRP laminate is shown in Fig. 6. The fourth beam (B3) is also with column stump but the orientation of CFRP laminate (as shown in Fig. 7) is different from that of third beam. The performances of these beams are described in the following sections.

The failure loads of all the beams are presented in Table 2. From the table it is seen that, the control beam has the lowest failure load (76 kN) compared to that of strengthened beam. The beam B1 showed the highest failure load (124 kN).The beam B2 had the failure load of 116 kN whereas the beam B3 had the failure load of only 90 kN.The beam B1 showed the highest failure load because in this case the CFRP laminate is placed in the center of the beam (longitudinally) i.e. the eccentricity is zero and the column stump is absent in this case. For the case of beam B2, though the area and length of CFRP laminate is same, there is some eccentricity of CFRP laminate and the restraint caused by the column stump is considered in this case which may led to exhibit a little bit less failure load than that of beam B1. But, still the failure load is high enough compared to control beam and the difference in failure load between B1 and B2 is very small. In the case of beam B3 the failure load is only 90 kN. In this case, the CFRP are connected to all four sides of the column stump and due to stress concentration at the end of one of the CFRP connection led to the premature failure of the beam. From this discussion it is clear that the CFRP orientation 2 (Fig. 6) is very effective, efficient and easy, and this orientation has overcome the constraint caused by the column in applying the CFRP laminate.

The control beam (B0) failed in the traditional flexure mode, because the beam was designed as under reinforced condition (Fig. 8).The beams B1 and B2 exhibited 'end peeling' while beam B3 exhibited premature failure (Figs. 9 to 11). In the case of B3, due to high stress concentration, the debonding occurred at the end of one of the CFRP connections which led to premature failure without reaching the full capacity. In the case of beam B2 and B3, as there is no end anchors provided, the failure initiated from the end of the strengthening plate and gradually it moved horizontally towards the center of the beam.

The load – deflection behavior of the beams B0, B1, B2, B3 are shown in Fig. 12. Linear increment of deflection was shown by all the beams before failure. The strengthened beams (B1, B2, B3) exhibited lower deflection than that of control beam because of having higher stiffness compared to control beam. It is also seen that the deflection at failure load of control beam is more than that of strengthened beams. The reason is that, the strengthened beams failed by plate debonding with brittle failure mode without any warning, whereas the control beam failed by flexure with ductile failure mode. The same deflection of the beam B1 and B2 at failure load is noticed.

The bar strains of the beams B0, B1, B2 and B3 at different loadings are shown in Fig. 13. It is seen that the bar strain of the control beam is more than that of others. The strain characteristics of the beam B1 and B2 are identical. From the figure it is noticed that there is a sudden strain increasing rate of beams at 15-25 kN loadings .This is due to the occurrence of 1st crack in the beams. Due to crack, the concrete released stresses to steel. Since it is a sudden release of stress, it acted as an impact stress on the steel bar which led to sudden jump in the strain of the steel bar. The theoretical yield load of control beam is 70.8 kN but in experiment this load is found 63 kN and the bar strain at this load is 0.004. At the same load (63 kN) the bar strain in beam B1, B2 and B3 are 0.0019, 0.0019 and 0.0023respectively. This variation in bar strain is due to the difference of the stiffness of the beams. The theoretical yield load of strengthened beam is 99 kN. For beams B1, B2, B3 the yield load was found 102,102 and 80 kN respectively and the bar strain at this load is 0.004, 0.004 and 0.004 respectively. It is found that, the theoretical yield load and the experimental yield load of the beam B1 and B2 are identical.

Fig. 14 shows the load versus concrete compressive strain at the top of the beam. It is seen that the strain of the control beam is more than that of strengthened beam. It is noticed that no beam has concrete compressive strain more than 0.0035, which indicate that the beams did not failed by concrete crushing. Pattern of strain changes with loading of the beams B1 and B2 are almost identical.

The strain was taken from demec points. It is seen that the strain of the beam B0 is higher than that of strengthened beams due to the variation in the stiffness. It is also noticed that, the strain variation characteristics of beam B1 and B2 are almost identical.

The CFRP laminate strain of strengthened beams is shown in Fig. 15. From the figure it is seen that, CFRP laminate don’t have definite yield point due to their elastic property. It is also seen that, The CFRP strain of beams B1 and B2 have the same characteristics. Due to premature debonding failure, the strain of CFRP of beam B3 is very less.

The 1st cracking loads of all the beams are shown in Table 2. The control beam depicts the lower 1st cracking load than that of strengthened beams. Beams B1, B2 and B3 showed the 1st cracking loads which are very close. It was due to similar material properties of concrete and strengthening laminate. The crack widths at 63 kN load of all the beams are shown in Table 2. The strengthened beam had shown the lower crack width than that of control beam.

Table 2. Test result

Beam name	1st crack load (kN)	1st crack load increase over control beam (%)	Failure load (kN)	Failure load increase over control beam (%)	Strain (micro) at 63 kN load			Concrete Compressive Strain (με) at failure	Mid-span deflection (mm) at 63 kN load	Crack width(mm) at 63 kN load	Mode of failure
					Steel bar strain (με)	Plate strain (με)	Concrete Compressive Strain (με)				
B0	19	-	76	-	4000	-	1600	2715	12	2.86	Flexure
B1	28	47	124	63	2000	2450	679	2478	9	0.6	End peeling
B2	21	16	116	54	2000	2200	787	2142	9	0.615	End peeling
B3	28	47	90	18	2200	731	940	2100	9	0.889	De bonding

5. FAILURE MODES

Fig. 8. Failure mode of beam B 0

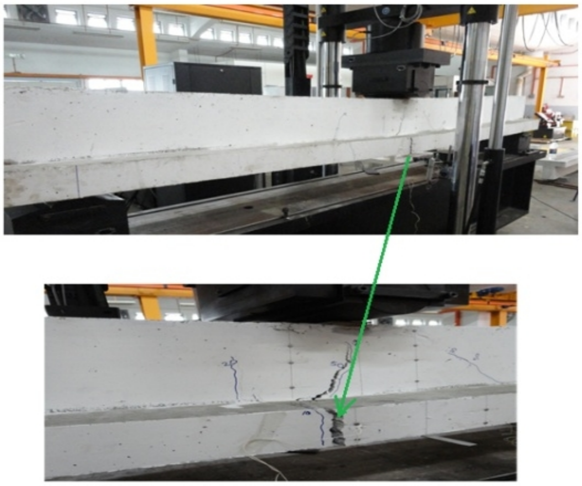


Fig. 9. Failure mode of beam B 1

Fig. 10. Failure mode of beam B 2

Fig. 11. Failure mode of beam B 3

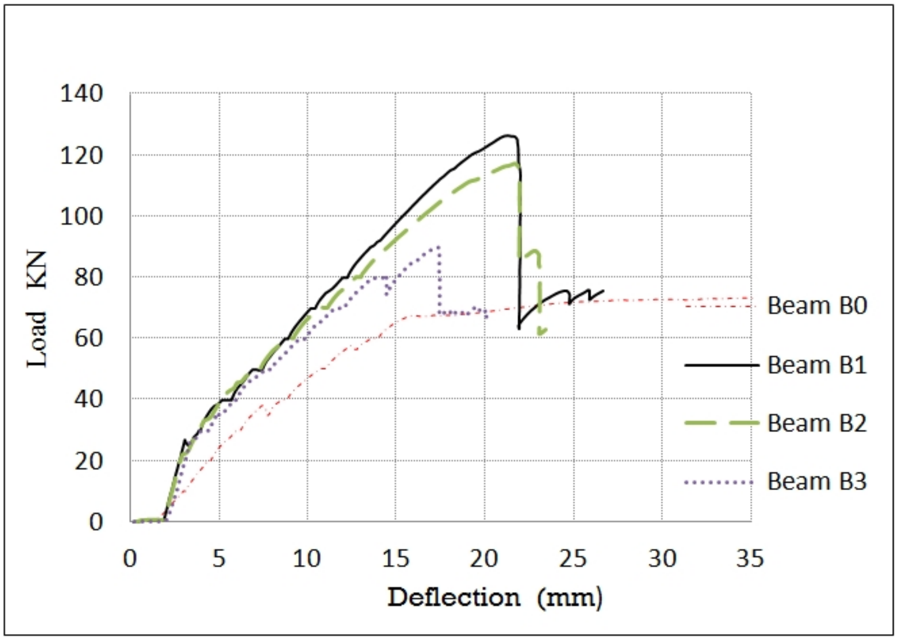


Fig. 12. Load versus mid span Deflection

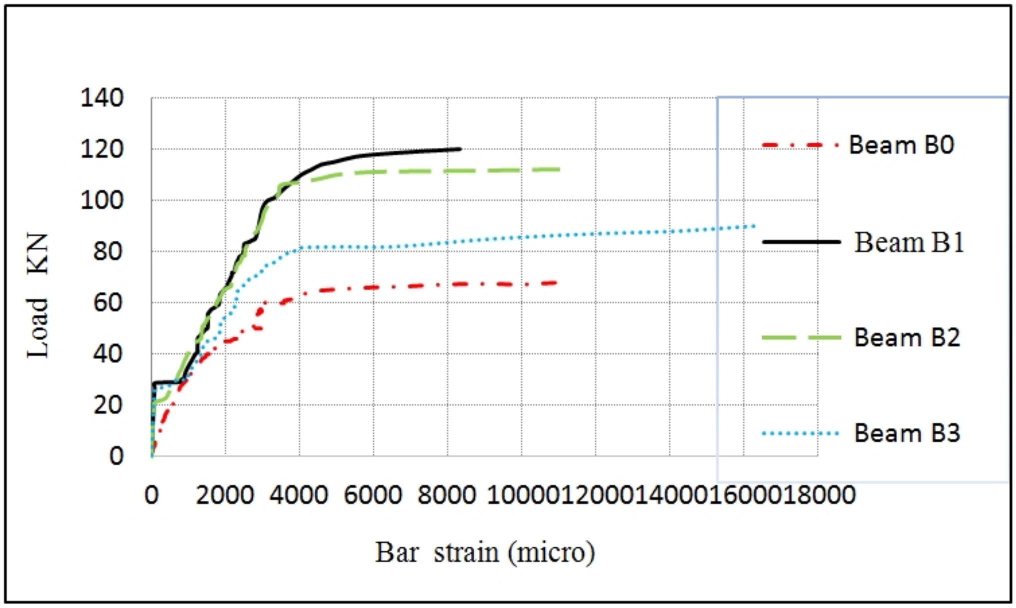


Fig. 13. Load versus bar strain

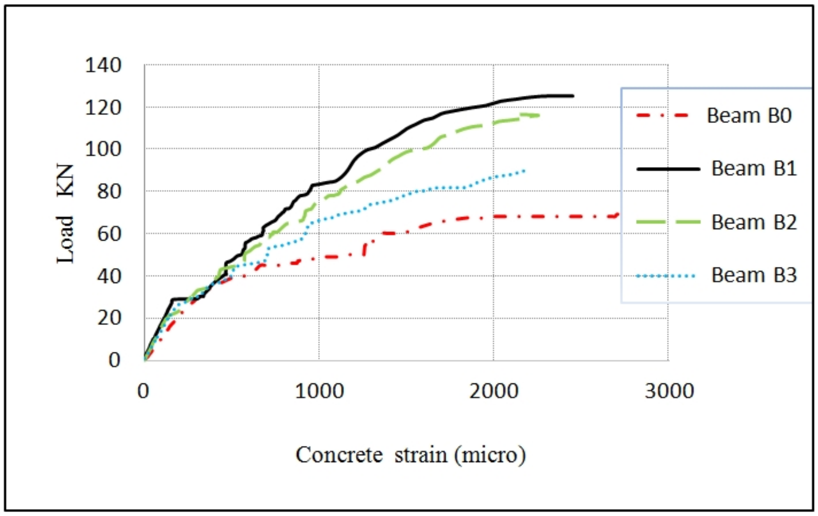


Fig. 14. Load versus concrete strain

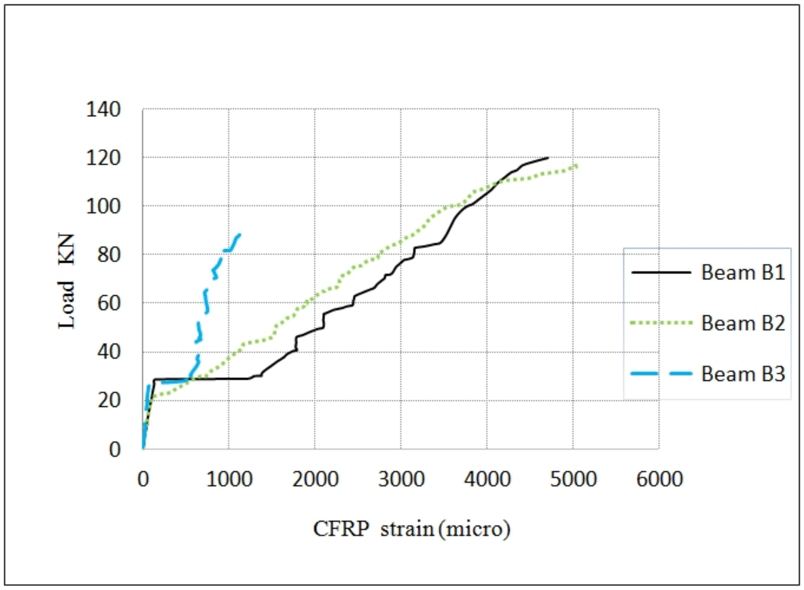


Fig. 15. Load versus CFRP strain

6. CONCLUSIONS

The following outcomes were concluded from this study:

I. Of the three orientations considered for strengthening of RC T-beams, Orientation 1 showed 38% higher strength and Orientation 2 showed 29% higher strength as compared to Orientation 3. Orientation 1 and Orientation 2 showed similar deflection and strain characteristics for concrete, steel bar & CFRP laminate. In Orientation 2, the restraint offered to the application of CFRP laminate due to the presence of column stump did not affect the strength of the beam significantly. The load carrying capacity was increased by about 50% over un-strengthened beam.

II. About 70 % load carrying capacity was increased over un-strengthened beam by strengthening both the tension and compression zone. Therefore, CFRP strengthening at both compression and tension zone is able to increase strength significantly.
III. The un-strengthened beam revealed the conventional flexural failure mode while end peeling of the CFRP laminate was the dominant mode of failure for all the strengthened beams tested.
IV. All strengthened beams had shown higher cracking and failure loads, less deflections, smaller crack widths and lower strain characteristics compared to that of un-strengthened control beams.
V. A good agreement was observed between the numerical and experimental results in terms of bar yield load, ultimate failure load, mid span deflection, bar strain, concrete strain and plate strain of all the beams.

RECOMMENDATIONS

The following recommendations are presented in order to develop and improve current findings:

I. The strengthened beams failed by end peeling. Providing end anchors may prevent this type of failure and can further improve the strength of beam. So, the implementation of end anchors along with designing their appropriate dimensions are strongly recommended for further research for strengthening such kind of beams.
II. The beams were tested under static loading condition only. More research is needed to determine the effect of repeated loading on strengthened beams.
III. The experiments were conducted only for flexure. The combined effect of strengthening both for flexure and shear can be of further interest.
IV. Mechanical fastened system, Near Surface Mounted (NSM) method of strengthening of such types of beams may be of particular interest.

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COMPETING INTERESTS

Authors have declared that no competing interests exist.

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APPENDIX A

A.1 Data Required for Design of Beam

The strengthened beams are designed in accordance with simplified stress block methods of BS EN 1992-1-1:(2004).The design procedures are described below:

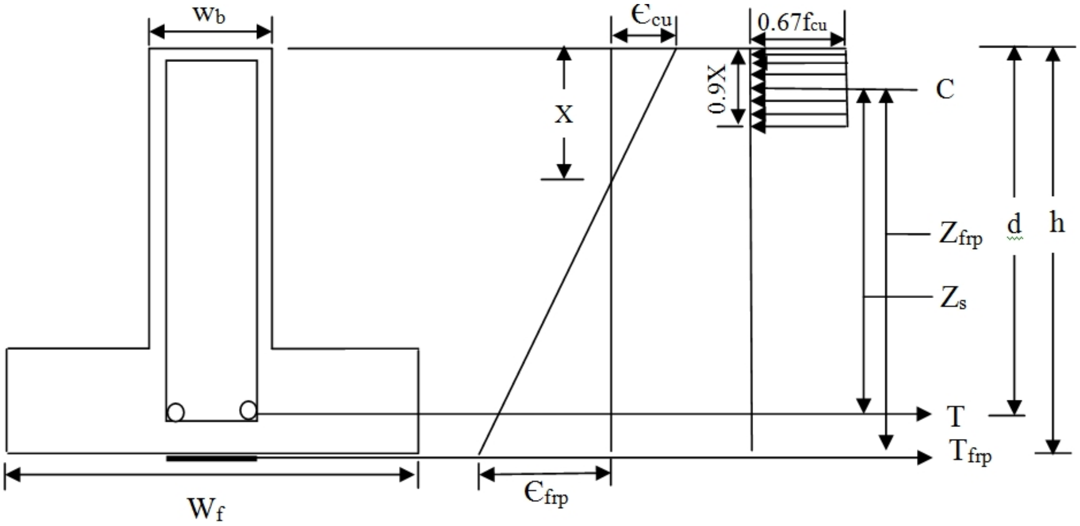

Fig. A1. **Stress block diagram of strengthened beam**

Strain of CFRP laminate = ϵ_{frp}
Depth of the beam, h = 325 mm
Effective depth of the beam, d = 268 mm
Width of the web, w_b = 150 mm
Width of the flange, w_f = 380 mm
Span length = 3000 mm
Moment arm of the steel bar, Z_s= (d - 0.9x/2)
Moment arm of the CFRP laminate, Z_{frp}= (h - 0.9x/2)
Cross sectional area of steel bar, A_s= 401.92 mm^2
Cross sectional area of CFRP plate = A_{frp}
Yield strength of steel bar, f_y= 560 MPa
Modulus of elasticity of CFRP laminate, E_{frp}=165 GPa
Tensile strength of steel bar, f_t = 645 MPa

A.2 Design of CFRP Laminate Strengthened Beam

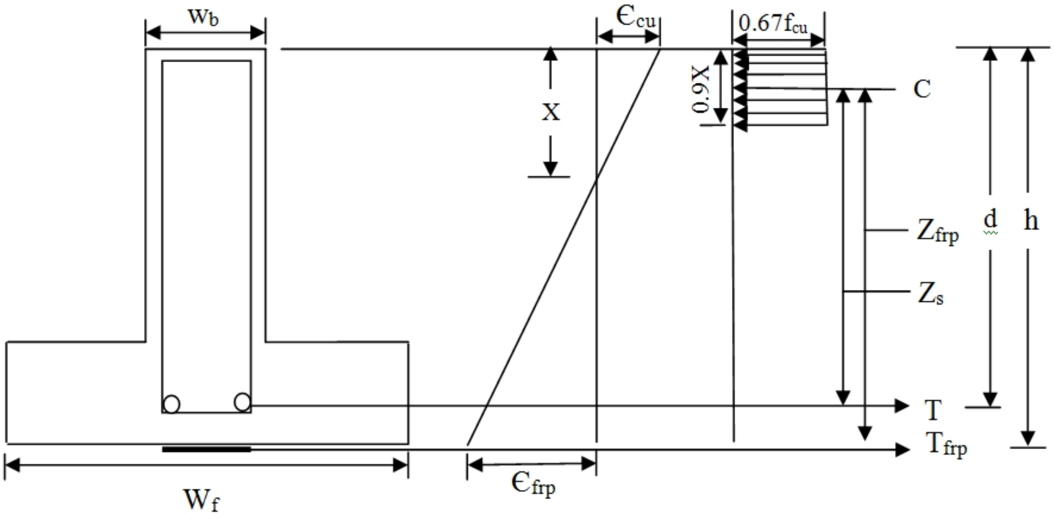

Fig. A2. **Stress block diagram of strengthened beam**

A.2.1 Depth of neutral axis

In accordance with BS EN 1992-1-1 :(2004), the design strain of concrete is 0 .0035.

According to Technical Report 55 (TR 55), the FRP strain should be less than 0.006 to avoid debonding failure.

From the Fig. A2,

$$\frac{0.0035}{0.006}=\frac{x}{h-x}\text{ --------------------------------- }(A.1)$$

$$x=0.368\times h$$

$$x=0.368\times 325=119.6mm$$

A.2.2 Required area of CFRP laminate

$$C=0.67\times f_{cu}\times 0.9x\times b_w\text{ ------------------------------ }(A.2)$$

$$C=0.603\times f_{cu}\times x\times b_w\text{ -------------------- }(A.3)$$

$$T_s+T_{frp}=C\text{ ------------------------ }(A.4)$$

$$T_{frp}=C-T_s\text{ ---------------------------- }(A.5)$$

$$T_s=A_s\times f_y\text{ ---------------------------- }(A.6)$$

$$\therefore T_{frp} = (0.603 \times f_{cu} \times x \times b_w) - (A_s \times f_y) ------------ (A.7)$$

$$E_{frp} = \frac{\sigma_{frp}}{\in_{frp}} ---------------------------------- (A.8)$$

$$\sigma_{frp} = E_{frp} \times \in_{frp} ---------------------------------- (A.9)$$

$$A_{frp} = \frac{T_{frp}}{\sigma_{frp}} = \frac{(0.603 \times f_{cu} \times x \times b_w) - (A_s \times f_y)}{E_{frp} \times \in_{frp}} ------------------ (A.10)$$

$$A_{frp} = \frac{(0.603 \times f_{cu} \times x \times b_w) - (A_s \times f_y)}{E_{frp} \times \in_{frp}} ------------------ (A.12)$$

$$A_{frp} = \frac{(0.603 \times 35 \times 119.6 \times 150) - 401.92 \times 560}{165000 \times 0.006} = 155.09\, mm^2$$

As 140 mm^2 (100 mm x 1.4 mm) is the maximum size available in the market, we can use (100 mm x 1.4 mm) size CFRP laminate.

A.3 Calculation of Bar Yield Load of Control Beam

The theoretical bar yield load of the control beam is calculated as followings:
From the Fig. A2,

$$T_s = C = 0.67 f_{cu}(0.9xb) = A_s f_y ---------------------------- (A.13)$$

Depth of neutral axis,

$$x = \frac{A_s f_y}{0.603 f_{cu} b} ---------------------------------- (A.14)$$

$$= \frac{401.92 \times 560}{0.603 \times 35 \times 150} = 71.09\, mm$$

$$M_{cy} = T_s(d - 0.45x) = A_s f_y (d - 0.45x) ---------------------- (A.15)$$

Bar yield load of control beam,

$$p_{yield} = \frac{2M_{cy}}{1.5} = \frac{2A_s f_y (d - 0.45x)}{1.5} ------------------------ (A.16)$$

$$p_{yield} = \frac{2 \times 401.92 \times 560(268 - 0.45 \times 71.09)}{1.5 \times 1000} = 70.8\, KN$$

A.4 Flexural Failure Load of Control Beam

From the Fig. A2

$$T_s = C = 0.67 f_{cu}(0.9xb) = A_s f_t \text{ ---------------------------- } (A.17)$$

$$x = \frac{A_s f_t}{0.603 f_{cu} b} \text{ ------------------------------------- } (A.18)$$

$$= \frac{401.92 \times 645}{0.603 \times 35 \times 150} = 81.88mm$$

$$M_{ct} = T_s(d - 0.45x) = A_s f_t (d - 0.45x) \text{ ---------------------- } (A.19)$$

$$p_{failure} = \frac{2M_{ct}}{1.5} = \frac{2A_s f_t (d - 0.45x)}{1.5} \text{ -------------------------- } (A.20)$$

$$p_{failure} = \frac{2 \times 401.92 \times 645(268 - 0.45 \times 81.88)}{1.5 \times 1000} = 79.80KN$$

A.5 Bar Yield Load of Strengthened Beam

Bar yield strain, $\varepsilon_s = \frac{Yieldstress}{ModulusofElasticity} = \frac{f_y}{E_s}$

The CFRP laminate strain of strengthened beam at bar yield can be obtained by trial and error.

Assume, x=0.5d =0.5 x 268 =134 mm

From the Fig. A2,

$$\in_{frp} = \frac{\in_s (h-x)}{d-x} = \frac{\in_s (h-.5d)}{d-.5d} = \frac{f_y(h-.5d)}{.5dE_s} \text{ ---------------------- } (A.21)$$

$$\sigma_{frp} = \in_{frp} E_{frp} = \frac{f_y(h-.5d)E_{frp}}{.5dE_s} \text{ --------------------------- } (A.22)$$

$$T = T_{frp} + T_s = \frac{f_y(h-.5d)E_{frp}A_{frp}}{.5dE_s} + A_s f_y \text{ --------------------- } (A.24)$$

$$T_{frp} = \sigma_{frp} A_{frp} = \frac{f_y (h-.5d) E_{frp} A_{frp}}{.5dE_s} ----------------------------(A.23)$$

$$C = 0.603 f_{cu} bx = T = \frac{f_y (h-.5d) E_{frp} A_{frp}}{.5dE_s} + A_s f_y ------------------(A.25)$$

$$x = \frac{1}{0.603 f_{cu} b} \left(\frac{f_y (h-.5d) E_{frp} A_{frp}}{.5dE_s} + A_s f_y \right) ----------------------(A.26)$$

$$= \frac{1}{0.603 \times 35 \times 150} \left[\frac{560 \times (325-134) \times 165000 \times 140}{134 \times 200000} + 401.92 \times 560 \right]$$

$$= 100.21 mm$$

Moment at yield of internal bar

$$M_{yc} = T_s Z_s + T_{frp} Z_{frp} ----------------------------------(A.27)$$

$$= A_s f_y (d - 0.45x) + \frac{f_y (h-x) E_{frp} A_{frp}}{E_s (d-x)} (h - 0.45x) ------------------(A.28)$$

$$= \frac{2}{1.5} [401.92 \times 560 \times (268 - 0.45 \times 100) + \frac{560 \times (325-100) \times 165 \times 140}{200 \times (268-100)} (325 - 0.45 \times 100)$$

$$= 99.26 KN$$

A.6 Failure Load of Strengthened Beam

$$\in_{frp} = \frac{0.0035(h-x)}{x} ----------------------------------(A.29)$$

$$\sigma_{frp} = \in_{frp} E_{frp} = \frac{0.0035(h-x)}{x} E_{frp} ----------------------------(A.30)$$

$$T_s = A_s f_t --(A.31)$$

$$T_{frp} = \sigma_{frp} A_{frp} --------------------------------------(A.32)$$

$$T = T_{frp} + T_s = \sigma_{frp} A_{frp} + A_s f_t ------------------------------(A.33)$$

$$C = 0.603 f_{cu} bx = T = \sigma_{frp} A_{frp} + A_s f_t --------------------------(A.34)$$

$$0.603 f_{cu} bx = A_s f_t + \frac{0.0035(h-x)}{x} E_{frp} A_{frp} ----------------------(A.35)$$

$$(0.603 f_{cu} b)x^2 - (A_s f_t)x = 0.0035(h-x)E_{frp}A_{frp} -----------------(A.36)$$

$$(0.603 f_{cu} b)x^2 - (A_s f_t - 0.0035 A_{frp} E_{frp})x - 0.0035 A_p E_{frp} h = 0 ---------(A.37)$$

$$x = \frac{-m \pm \sqrt{m^2 - 4nl}}{2l} ----------------------------------(A.38)$$

$$l = 0.603 f_{cu} b = 0.603 \times 35 \times 150 = 3165.75$$

$$m = -(A_s f_t - 0.0035 A_{frp} E_{frp}) = -(401.92 \times 645 - 0.0035 \times 140 \times 165000) = -178388.4$$

$$n = -0.0035 A_p E_{frp} h = -0.0035 \times 140 \times 165000 \times 325 = -26276250$$

$$x = \frac{1783884 \pm \sqrt{(-1783884)^2 - 4 \times (-26276250)} \times 3165.75}{2 \times 3165.75}$$

$$x = 123.53$$

And,

$$M_{tc} = T_s Z_s + T_p Z_{frp} = A_s f_t (d - 0.45x) + A_{frp} \sigma_{frp} (h - 0.45x) ------------(A.39)$$

CFRP laminate strain,

$$\in_{frp} = \frac{0.0035(h-x)}{x} = \frac{0.0035 \times (325 - 123.53)}{123.53} = 0.0057 -----------------(A.40)$$

$$p_{ut} = \frac{2M_{tc}}{1.5} = \frac{2}{1.5}[A_s f_t (d - 0.45x) + A_{frp} \sigma_{frp} (h - 0.45x)] -----------------(A.41)$$

Putting the value of x,

$$= \frac{2}{1.5 \times 1000}[401.92 \times 645(268 - 0.45 \times 123.53) + 140 \times 165000 \times 0.0057 \times (325 - 0.45 \times 123.53)]$$

$$= 120 \ KN$$

Tensile strength of steel bar, f_t = 645 MPa

Peer-review history:
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