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The Representations by Certain Duodenary Quadratic Forms

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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Abstract

The determination of the number of representations of a positive integer by certain quadratic forms is an important goal of number theory. Formulae for $N(1^{2i}, 2^{2j}, 3^{2k}, 6^{2l}; n)$ for the **nine octonary** quadratic forms appear in the literature, whose coefficients are $1, 2, 3$ and 6. Here, we determine formulae, for the numbers of representations of a positive integer by **one hundred and six different duodenary** quadratic forms whose coefficients are $1, 2, 3$ and 6.

Keywords: Duodenary quadratic forms; representations; theta functions; Dedekind eta function; Eisenstein series; Eisenstein forms; modular forms; cusp forms.

Mathematics Subject Classification 2010: 11E25, 11E20, 11F11, 11F20, 11F27.

1 Introduction

It is interesting and important to determine explicit formulas of the representation number of positive definite quadratic forms.

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The work on representation number $N(1,1,n)$ of quadratic forms has been started by Fermat in 1640 on x^2+y^2. It would be nice to obtain such simple formulas for other positive definite quadratic forms so that we would be able to understand the number of solutions of the equation $Q=n$ for any positive integer n.

Later the formula

$$N(1,1,n)=4\left(\sum_{d|n,d \text{ is odd}}(-1)^{\frac{d-1}{2}}\right)$$

has been proved by Euler. First systematic treatment of binary quadratic forms is due to Legendre. Afterwards it was advanced by Jacobi, with the proof of

$$N(1,1,1,1,n)=8\left(\sum_{d|n,4\nmid d} d\right) \text{ for } x^2+y^2+z^2+t^2.$$

The theory was advanced much further by Gauss in Disquisitiones Arithmetica. The research of Gauss strongly influenced both the arithmetical theory of quadratic forms in more than two variables and subsequent development of algebraic number theory. Since then, there are many more representation number formulas obtained for quadratic forms. Especially, by means of the deep theorems of Hecke [1] and Schoeneberg [2], Modular Forms have been used in the representation number of several quadratic forms. The generalized theta series ([3],[4], [5]), quasimodular forms ([6],[7],[8], [9]). and several other methods have been also used for the representation number formulae.

The divisor function $\sigma_i(n)$ is defined for a positive integer i by

$$\begin{aligned}\sigma_i(n) \; &:= \sum_{d \text{ positive integer}, d|n} d^i, \text{if } n \text{ is a positive integer, and} \qquad (1.1)\\ \sigma_i(n) \; &:= 0 \text{ if n is not a positive integer.}\end{aligned}$$

The Dedekind eta function and the theta function are defined by

$$\eta(z):=q^{1/24}\prod_{n=1}^{\infty}(1-q^n), \varphi(q):=\sum_{n\in\mathbb{Z}}q^{n^2} \qquad (1.2)$$

where

$$q:=e^{2\pi iz}, z\in H=\{x+iy: y>0\} \qquad (1.3)$$

and an eta quotient of level N is defined by

$$f(z):=\prod_{m|N}\eta(mz)^{a_m}, N,m\in\mathbb{N}, a_m\in\mathbb{Z}. \qquad (1.4)$$

Here we give the following Lemma, see[[10] Theorem 1.64] about the modularity of an eta quotient.

Lemma 1 *An eta quotient of level N is a meromorphic modular form of weight $\frac{1}{2}\sum_{m|N}a_m$ on $\Gamma_0(N)$ having rational coefficients with respect to q if*

$$a) \sum_{m|N}a_m \text{ is even,}$$

$$b) \sum_{m|N}ma_m \equiv \sum_{m|N}\frac{N}{m}a_m \equiv 0 \text{ mod } 24,$$

$$c) \prod_{m|N}m^{a_m} \text{ is a square in } \mathbb{Q}.$$

For $a_1, ..., a_{12} \in \mathbb{N}$ and a nonnegative integer n, we define

$$N(a_1, ..., a_{12}; n) := card\{(x_1, ..., x_{12}) \in \mathbb{Z}^{12} | n = a_1 x_1^2 + \cdots + a_{12} x_{12}^2\}.$$

Clearly $N(a_1, ..., a_{12}; 0) = 1$ and without loss of generality we can assume that

$$a_1 \leq ... \leq a_{12}.$$

Now let's consider duodenary quadratic forms of the form

$$\begin{aligned} Q \quad : \quad & = x_1^2 + \cdots + x_a^2 + 2(x_{a+1}^2 + \cdots + x_{a+b}^2) + 3(x_{a+b+1}^2 + \cdots + x_{a+b+c}^2) \\ & +6(x_{a+b+c+1}^2 + \cdots + x_{a+b+c+d=12}^2), \end{aligned}$$

where, $a, b, c \in \mathbb{Z}$,$0 \leq a \leq 12$, $0 \leq b \leq 12, 0 \leq c \leq 12, 0 \leq d \leq 12$.

Table 1

$(0,2,2,8)$	$(1,1,1,9)$	$(2,0,0,10)$	$(3,3,1,5)$	$(3,7,1,1)$	$(4,8,0,0)$	$(6,4,2,0)$
$(0,2,4,6)$	$(1,1,3,7)$	$(2,0,2,8)$	$(3,3,3,3)$	$(4,0,0,8)$	$(5,1,1,5)$	$(6,6,0,0)$
$(0,2,6,4)$	$(1,1,5,5)$	$(2,0,4,6)$	$(3,3,5,1)$	$(4,0,2,6)$	$(5,1,3,3)$	$(7,1,1,3)$
$(0,2,8,2)$	$(1,1,7,3)$	$(2,0,6,4)$	$(3,5,1,3)$	$(4,0,4,4)$	$(5,1,5,1)$	$(7,1,3,1)$
$(0,2,10,0)$	$(1,1,9,1)$	$(2,0,8,2)$	$(3,5,3,1)$	$(4,0,6,2)$	$(5,3,1,3)$	$(7,3,1,1)$
$(0,4,2,6)$	$(1,3,1,7)$	$(2,0,10,0)$	$(2,2,8,0)$	$(4,0,8,0)$	$(5,3,3,1)$	$(8,0,0,4)$
$(0,4,4,4)$	$(1,3,3,5)$	$(2,2,0,8)$	$(2,4,0,6)$	$(4,2,0,6)$	$(5,5,1,1)$	$(8,0,2,2)$
$(0,4,6,2)$	$(1,3,5,3)$	$(2,2,2,6)$	$(2,4,2,4)$	$(4,2,2,4)$	$(6,0,0,6)$	$(8,0,4,0)$
$(0,4,8,0)$	$(1,3,7,1)$	$(2,2,4,4)$	$(2,4,4,2)$	$(4,2,4,2)$	$(6,0,2,4)$	$(8,2,0,2)$
$(0,6,2,4)$	$(1,5,1,5)$	$(2,2,6,2)$	$(2,4,6,0)$	$(4,2,6,0)$	$(6,0,4,2)$	$(8,2,2,0)$
$(0,6,4,2)$	$(1,5,3,3)$	$(2,10,0,0)$	$(2,6,0,4)$	$(4,4,0,4)$	$(6,0,6,0)$	$(8,4,0,0)$
$(0,6,6,0)$	$(1,5,5,1)$	$(3,1,1,7)$	$(2,6,2,2)$	$(4,4,2,2)$	$(6,2,0,4)$	$(9,1,1,1)$
$(0,8,2,2)$	$(1,7,1,3)$	$(3,1,3,5)$	$(2,6,4,0)$	$(4,4,4,0)$	$(6,2,2,2)$	$(10,0,0,2)$
$(0,8,4,0)$	$(1,7,3,1)$	$(3,1,5,3)$	$(2,8,0,2)$	$(4,6,0,2)$	$(6,2,4,0)$	$(10,0,2,0)$
$(0,10,2,0)$	$(1,9,1,1)$	$(3,1,7,1)$	$(2,8,2,0)$	$(4,6,2,0)$	$(6,4,0,2)$	$(10,2,0,0)$
						$(12,0,0,0)$

We write $N(1^a, 2^b, 3^c, 6^d; n)$ to denote the number of representations of n by a duodenary quadratic form (a, b, c, d). Its theta function is obviously

$$\Theta_Q = \varphi^a(q)\varphi^b(q^2)\varphi^c(q^3)\varphi^d(q^6).$$

Formulae for $N(1^{2i},\ 2^{2j},\ 3^{2k},\ 6^{2l};\ n)$ for the nine octonary quadratic forms $(2i, 2j, 2k, 2l) = (8,0,0,0), (2,6,0,0), (4,4,0,0), (6,2,0,0), (2,0,6,0), (4,0,4,0), (6,0,2,0), (4,0,0,4)$, and $(0,4,4,0)$ appear in the literature, [11],[12],[13],[14],[15] and [16]. Here, we will classify all fourtuples (a, b, c, d) for which Θ_Q is a modular form of weight 6 with level 24. Then we will obtain their representation numbers in terms of the coefficients of Eisenstein series and some eta quotients.

First, by the following Theorem, we characterize the facts that

$$\varphi^a(q)\varphi^b(q^2)\varphi^c(q^3)\varphi^d(q^6)$$

are in $M_6(\Gamma_0(24))$.

Theorem 2 Let

$$\begin{aligned} Q \quad : \quad & = x_1^2 + \cdots + x_a^2 + 2(x_{a+1}^2 + \cdots + x_{a+b}^2) + 3(x_{a+b+1}^2 + \cdots + x_{a+b+c}^2) \\ & +6(x_{a+b+c+1}^2 + \cdots + x_{a+b+c+d=12}^2) \end{aligned}$$

where, $a,b,c\in\mathbb{Z}, 0\le a\le 12,\ 0\le b\le 12, 0\le c\le 12, 0\le d\le 12$, be a duodenary quadratic form. Then its theta series is of the form

$$\begin{aligned}\Theta_Q &= \varphi^a(q)\,\varphi^b\left(q^2\right)\varphi^c\left(q^3\right)\varphi^d\left(q^6\right)=\eta^{-2a}(q)\,\eta^{5a-2b}\left(q^2\right)\eta^{-2c}\left(q^3\right)\\ &\quad \eta^{5b-2a}\left(q^4\right)\eta^{5c-2d}\left(q^6\right)\eta^{-2b}\left(q^8\right)\eta^{5d-2c}\left(q^{12}\right)\eta^{-2d}\left(q^{24}\right).\end{aligned}$$

Moreover, it is in $M_6(\Gamma_0(24))$ if and only if (a,b,c,d) is given in the Table 1. Here we also see that a,b,c,d are either both even or both odd.

Proof. It follows from the Lemma 1, holomorphicity criterion in [[17] Corollary 2.3,p.37] and the fact that

$$\varphi(q)=\frac{\eta^5\left(q^2\right)}{\eta^2(q)\,\eta^2\left(q^4\right)}.$$

The condition

$$1^{a_1}2^{a_2}3^{a_3}4^{a_4}6^{a_6}8^{a_8}12^{a_{12}}24^{a_{24}}$$

is a square of a rational number implies that either a,b,c,d are both even or both odd integers. □

Now let,

$E_6(q)=1-504\sum_{n=1}^{\infty}\sigma_5(n)\,q^n$,

$A_1(q):=\eta(z)^6\,\eta(3z)^6=q\prod_{n=1}^{\infty}(1-q^n)^6\left(1-q^{3n}\right)^6$,

$A_2(q):=\eta(2z)^{12}=q\prod_{n=1}^{\infty}\left(1-q^{2n}\right)^{12}$,

$A_3(q):=\frac{\eta(2z)^9\eta(3z)^9}{\eta(z)^3\eta(6z)^3}=q\prod_{n=1}^{\infty}\frac{\left(1-q^{2n}\right)^9\left(1-q^{3n}\right)^9}{(1-q^n)^3\left(1-q^{6n}\right)^3}$,

$A_4(q):=\frac{\eta(2z)^{10}\eta(4z)^{10}}{\eta(z)^4\eta(8z)^4}=q\prod_{n=1}^{\infty}\frac{\left(1-q^{2n}\right)^{10}\left(1-q^{4n}\right)^{10}}{(1-q^n)^4\left(1-q^{8n}\right)^4}$,

$A_5(q):=\frac{\eta(2z)^3\eta(3z)^2\eta(4z)\eta(6z)\eta(8z)^2\eta(24z)^6}{\eta(z)^2\eta(12z)^1}$

$=q^7\prod_{n=1}^{\infty}\frac{\left(1-q^{2n}\right)^3\left(1-q^{3n}\right)^2\left(1-q^{4n}\right)\left(1-q^{6n}\right)\left(1-q^{8n}\right)^2\left(1-q^{24n}\right)^6}{(1-q^n)^2\left(1-q^{12n}\right)}$,

$A_6(q):=\frac{\eta(2z)^3\eta(3z)^2\eta(4z)^3\eta(6z)\eta(24z)^{10}}{\eta(z)^2\eta(8z)^2\eta(12z)^3}$

$=q^9\prod_{n=1}^{\infty}\frac{\left(1-q^{2n}\right)^3\left(1-q^{3n}\right)^2\left(1-q^{4n}\right)^3\left(1-q^{6n}\right)\left(1-q^{24n}\right)^{10}}{(1-q^n)^2\left(1-q^{8n}\right)^2\left(1-q^{12n}\right)^3}$,

$A_7(q):=\frac{\eta(2z)\eta(3z)^3\eta(4z)^2\eta(12z)^2\eta(24z)^8}{\eta(z)\eta(6z)^3}$

$=q^9\prod_{n=1}^{\infty}\frac{\left(1-q^{2n}\right)\left(1-q^{3n}\right)^3\left(1-q^{4n}\right)^2\left(1-q^{12n}\right)^2\left(1-q^{24n}\right)^8}{(1-q^n)\left(1-q^{6n}\right)^3}$.

Theorem 3 The set

$$\begin{aligned}&\{E_6, E_6(2z), E_6(3z), E_6(4z), E_6(6z), E_6(8z), E_6(12z), E_6(24z), A_1, A_1(2z), A_1(4z)\\ A_1(8z),\ &A_2, A_2(2z), A_2(3z), A_2(6z), A_3, A_3(2z), A_3(4z), A_4, A_4(3z), A_5, A_6, A_7\}\end{aligned}$$

is a basis of $M_6(\Gamma_0(24))$. Moreover, the unique newform in $S_6(\Gamma_0(3))$ is A_1,the unique newform in $S_6(\Gamma_0(4))$ is A_2, the unique newform in $S_6(\Gamma_0(6))$ is

$$\begin{aligned}\Delta_{6,6}(z) &= A_3(z)+\eta(z)^9\,\eta(2z)^{-3}\,\eta(3z)^{-3}\,\eta(6z)^9\\ &= \frac{1}{4368}E_6(4z)-\frac{113}{4368}E_6(8z)+\frac{545}{4368}E_6(12z)+\frac{3935}{4368}E_6(24z)\\ &\quad -A_1(z)-8A_1(2z)+\frac{263}{26}A_1(8z)+\frac{1059}{13}A_1(4z)+2A_3(z)-10A_3(4z),\end{aligned}$$

the unique newform in $S_6\left(\Gamma_0\left(8\right)\right)$

$$\begin{aligned}
\Delta_{8,6}\left(z\right) &= A_4\left(z\right)-4\eta\left(z\right)^4\eta\left(2z\right)^2\eta\left(4z\right)^2\eta\left(8z\right)^4\\
&= \frac{1}{4368}E_6(4z)-\frac{113}{4368}E_6(8z)+\frac{545}{4368}E_6(12z)+\frac{3935}{4368}E_6(24z)\\
&\quad+\frac{263}{26}A_1(4z)+\frac{1059}{13}A_1(8z)-A_2(z)-8A_2(2z)-10A_3(4z)+2A_4(z),
\end{aligned}$$

the three unique newforms in $S_6\left(\Gamma_0\left(24\right)\right)$ are

$$\begin{aligned}
\Delta_{24,6,1}\left(z\right) &= \frac{1}{4368}E_6(4z)-\frac{113}{4368}E_6(8z)+\frac{545}{4368}E_6(12z)+\frac{3935}{4368}E_6(24z)\\
&\quad+\frac{7}{9}A_1(z)+\frac{10}{9}A_1(2z)+\frac{9109}{78}A_1(8z)+\frac{29801}{39}A_1(4z)-\frac{40}{3}A_2(2z)\\
&\quad+48A_2(3z)+168A_2(6z)-\frac{16}{9}A_3(z)+\frac{128}{9}A_3(2z)-\frac{542}{3}A_3(4z)+2A_4(z)\\
&\quad-54A_4(3z)+384A_5(z)-384A_5(z)+512A_6(z),
\end{aligned}$$

$$\begin{aligned}
\Delta_{24,6,2}\left(z\right) &= \frac{1}{4368}E_6(4z)-\frac{113}{4368}E_6(8z)+\frac{545}{4368}E_6(12z)+\frac{3935}{4368}E_6(24z)\\
&\quad-\frac{11}{9}A_1(z)-\frac{8}{9}A_1(2z)+\frac{20341}{78}A_1(4z)+\frac{89705}{39}A_1(8z)+\frac{32}{3}A_2(2z)\\
&\quad-60A_2(3z)-48A_2(6z)+\frac{2}{9}A_3(z)-\frac{232}{9}A_3(2z)-\frac{542}{3}A_3(4z)+2A_4(z)\\
&\quad+54A_4(3z)-192A_5(z)-960A_5(z)+512A_6(z),
\end{aligned}$$

$$\begin{aligned}
\Delta_{24,6,3}\left(z\right) &= \frac{1}{4368}E_6(4z)-\frac{113}{4368}E_6(8z)+\frac{545}{4368}E_6(12z)+\frac{3935}{4368}E_6(24z)\\
&\quad-\frac{7}{9}A_1(z)+\frac{57695}{234}A_1(4z)+\frac{49769}{39}A_1(8z)+\frac{4}{3}A_2(z)+\frac{32}{3}A_2(2z)\\
&\quad-48A_2(3z)-48A_2(6z)-\frac{14}{9}A_3(z)-\frac{56}{3}A_3(2z)-\frac{1882}{9}A_3(4z)\\
&\quad+2A_4(z)+54A_4(3z)+448A_5(z)-1344A_5(z).
\end{aligned}$$

Proof. $M_6(\Gamma_0(24))$ is 24 dimensional, $S_6(\Gamma_0(24))$ is 16 dimensional, see [18] (Chapter 3, pg.87 and Chapter 5, pg.197), and generated by

$$\begin{aligned}
&\Delta_{3,6}\left(z\right)=A_1(z), \Delta_{3,6}\left(2z\right), \Delta_{3,6}\left(4z\right), \Delta_{3,6}\left(8z\right),\\
&\Delta_{4,6}\left(z\right)=A_2(z), \Delta_{4,6}\left(2z\right), \Delta_{4,6}\left(3z\right), \Delta_{4,6}\left(6z\right),\\
&\Delta_{6,6}\left(z\right), \Delta_{6,6}\left(2z\right), \Delta_{6,6}(4z),\\
&\Delta_{8,6}\left(z\right), \Delta_{8,6}\left(3z\right),\\
&\Delta_{24,6,1}\left(z\right), \Delta_{24,6,2}\left(z\right), \Delta_{24,6,3}\left(z\right),
\end{aligned}$$

where $\Delta_{3,6}$ is the unique newform in $S_6(\Gamma_0(3))$; $\Delta_{4,6}$ is the unique newform in $S_6(\Gamma_0(4))$; $\Delta_{6,6}$ is the unique newform in $S_6(\Gamma_0(6))$, $\Delta_{8,6}$ is the unique newform in $S_6(\Gamma_0(8))$; and $\Delta_{24,6,1}, \Delta_{24,6,2}, \Delta_{24,6,3}$ are the unique newforms in $S_6(\Gamma_0(24))$. □

2 Conclusion

We have used Magma for the calculation of the coefficients in the Appendix. This work can be extended to other positive even weights, greater than 6.

Competing Interests

Author has declared that no competing interests exist.

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Appendix

The following formulae for the representation numbers are valid.

$$N(2^2,3^2,6^8;n) = -\frac{1}{366\,912}\sigma_5(n) + \frac{1}{366\,912}\sigma_5\left(\frac{n}{2}\right) + \frac{61}{122\,304}\sigma_5\left(\frac{n}{3}\right) + \frac{1}{11\,466}\sigma_5\left(\frac{n}{4}\right)$$
$$-\frac{61}{122\,304}\sigma_5\left(\frac{n}{6}\right) - \frac{32}{5733}\sigma_5\left(\frac{n}{8}\right) - \frac{61}{3822}\sigma_5\left(\frac{n}{12}\right) + \frac{1952}{1911}\sigma_5\left(\frac{n}{24}\right) + \frac{115}{273}a_1(n)$$
$$+\frac{2606}{819}a_1(\tfrac{n}{2}) + \frac{25\,360}{819}a_1(\tfrac{n}{4}) + \frac{92\,416}{273}a_1(\tfrac{n}{8}) - \frac{1}{24}a_2(n) - \frac{8}{3}a_2(\tfrac{n}{2}) + \frac{93}{8}a_2(\tfrac{n}{3})$$
$$+24a_2(\tfrac{n}{6}) - \frac{8}{21}a_3(n) + \frac{64}{9}a_3(\tfrac{n}{2}) - \frac{2368}{63}a_3(\tfrac{n}{4}) - 12a_4(\tfrac{n}{3}) + 32a_5(n) + 32a_6(n)$$
$$+128a_7(n),$$

$$N(2^2,3^4,6^6;n) = -\frac{1}{183\,456}\sigma_5(n) + \frac{1}{183\,456}\sigma_5\left(\frac{n}{2}\right) + \frac{61}{61\,152}\sigma_5\left(\frac{n}{3}\right) + \frac{1}{11\,466}\sigma_5\left(\frac{n}{4}\right)$$
$$-\frac{61}{61\,152}\sigma_5\left(\frac{n}{6}\right) - \frac{32}{5733}\sigma_5\left(\frac{n}{8}\right) - \frac{61}{3822}\sigma_5\left(\frac{n}{12}\right) + \frac{1952}{1911}\sigma_5\left(\frac{n}{24}\right) + \frac{230}{273}a_1(n)$$
$$+\frac{4484}{819}a_1\left(\frac{n}{2}\right) + \frac{15\,248}{273}a_1\left(\frac{n}{4}\right) + \frac{162\,304}{273}a_1\left(\frac{n}{8}\right) - \frac{1}{12}a_2(n) - \frac{20}{3}a_2\left(\frac{n}{2}\right)$$
$$+\frac{93}{4}a_2\left(\frac{n}{3}\right) + 36a_2\left(\frac{n}{6}\right) - \frac{16}{21}a_3(n) + \frac{112}{9}a_3\left(\frac{n}{2}\right) - \frac{512}{7}a_3\left(\frac{n}{4}\right) - 24a_4\left(\frac{n}{3}\right)$$
$$+64a_5(n) + 64a_6(n) + 256a_7(n),$$
$$N(2^2,3^6,6^4;n) = -\frac{1}{91\,728}\sigma_5(n) + \frac{1}{91\,728}\sigma_5\left(\frac{n}{2}\right) + \frac{61}{30\,576}\sigma_5\left(\frac{n}{3}\right) + \frac{1}{11\,466}\sigma_5\left(\frac{n}{4}\right)$$
$$-\frac{61}{30\,576}\sigma_5\left(\frac{n}{6}\right) - \frac{32}{5733}\sigma_5\left(\frac{n}{8}\right) - \frac{61}{3822}\sigma_5\left(\frac{n}{12}\right) + \frac{1952}{1911}\sigma_5\left(\frac{n}{24}\right) + \frac{1016}{819}a_1(n)$$
$$+\frac{7148}{819}a_1\left(\frac{n}{2}\right) + \frac{99\,616}{819}a_1\left(\frac{n}{4}\right) + \frac{116\,224}{91}a_1\left(\frac{n}{8}\right) - \frac{1}{6}a_2(n) - 12a_2\left(\frac{n}{2}\right)$$
$$+\frac{69}{2}a_2\left(\frac{n}{3}\right) + 36a_2\left(\frac{n}{6}\right) - \frac{68}{63}a_3(n) + \frac{160}{9}a_3\left(\frac{n}{2}\right) - \frac{9088}{63}a_3\left(\frac{n}{4}\right) - 36a_4\left(\frac{n}{3}\right)$$
$$+64a_5(n) + 64a_6(n) + 512a_7(n),$$
$$N(2^2,3^8,6^2;n) = -\frac{1}{45\,864}\sigma_5(n) + \frac{1}{45\,864}\sigma_5\left(\frac{n}{2}\right) + \frac{61}{15\,288}\sigma_5\left(\frac{n}{3}\right) + \frac{1}{11\,466}\sigma_5\left(\frac{n}{4}\right)$$
$$-\frac{61}{15\,288}\sigma_5\left(\frac{n}{6}\right) - \frac{32}{5733}\sigma_5\left(\frac{n}{8}\right) - \frac{61}{3822}\sigma_5\left(\frac{n}{12}\right) + \frac{1952}{1911}\sigma_5\left(\frac{n}{24}\right) + \frac{1304}{819}a_1(n)$$
$$+\frac{9928}{819}a_1\left(\frac{n}{2}\right) + \frac{219\,008}{819}a_1\left(\frac{n}{4}\right) + \frac{256\,000}{91}a_1\left(\frac{n}{8}\right) - \frac{1}{3}a_2(n) - 20a_2\left(\frac{n}{2}\right)$$
$$+45a_2\left(\frac{n}{3}\right) + 12a_2\left(\frac{n}{6}\right) - \frac{80}{63}a_3(n) + \frac{224}{9}a_3\left(\frac{n}{2}\right) - \frac{18\,944}{63}a_3\left(\frac{n}{4}\right) - 48a_4\left(\frac{n}{3}\right)$$
$$+1024a_7(n),$$

$$N(2^2,3^{10};n) = -\frac{1}{22\,932}\sigma_5(n) + \frac{1}{22\,932}\sigma_5\left(\frac{n}{2}\right) + \frac{61}{7644}\sigma_5\left(\frac{n}{3}\right)$$
$$+\frac{1}{11\,466}\sigma_5\left(\frac{n}{4}\right) - \frac{61}{7644}\sigma_5\left(\frac{n}{6}\right) - \frac{32}{5733}\sigma_5\left(\frac{n}{8}\right) - \frac{61}{3822}\sigma_5\left(\frac{n}{12}\right) + \frac{1952}{1911}\sigma_5\left(\frac{n}{24}\right)$$
$$+\frac{2062}{819}a_1(n) + \frac{13\,304}{819}a_1\left(\frac{n}{2}\right) + \frac{280\,160}{819}a_1\left(\frac{n}{4}\right) + \frac{1140\,736}{273}a_1\left(\frac{n}{8}\right) - \frac{92}{3}a_2\left(\frac{n}{2}\right)$$
$$+72a_2\left(\frac{n}{3}\right) - 180a_2\left(\frac{n}{6}\right) + \frac{92}{63}a_3(n) + \frac{400}{9}a_3\left(\frac{n}{2}\right) - \frac{24\,320}{63}a_3\left(\frac{n}{4}\right)$$
$$-4a_4(n) - 60a_4\left(\frac{n}{3}\right) - 512a_5(n) + 1024a_6(n) + 1536a_7(n),$$
$$N(2^4,3^2,6^6;n) = -\frac{1}{183\,456}\sigma_5(n) + \frac{1}{183\,456}\sigma_5\left(\frac{n}{2}\right) - \frac{5}{10\,192}\sigma_5\left(\frac{n}{3}\right) - \frac{1}{5733}\sigma_5\left(\frac{n}{4}\right)$$
$$+\frac{5}{10\,192}\sigma_5\left(\frac{n}{6}\right) + \frac{64}{5733}\sigma_5\left(\frac{n}{8}\right) - \frac{10}{637}\sigma_5\left(\frac{n}{12}\right) + \frac{640}{637}\sigma_5\left(\frac{n}{24}\right) - \frac{584}{819}a_1(n)$$
$$-\frac{3160}{819}a_1\left(\frac{n}{2}\right) - \frac{27\,424}{819}a_1\left(\frac{n}{4}\right) - \frac{91\,648}{273}a_1\left(\frac{n}{8}\right) + \frac{7}{12}a_2(n) + \frac{32}{3}a_2\left(\frac{n}{2}\right)$$
$$-\frac{39}{2}a_2\left(\frac{n}{3}\right) + \frac{8}{63}a_3(n) - \frac{32}{9}a_3\left(\frac{n}{2}\right) + \frac{2944}{63}a_3\left(\frac{n}{4}\right) + 36a_4\left(\frac{n}{3}\right) + 64a_5(n)$$
$$-192a_6(n) - 256a_7(n),$$
$$N(2^4,3^4,6^4;n) = -\frac{1}{91\,728}\sigma_5(n) + \frac{1}{91\,728}\sigma_5\left(\frac{n}{2}\right) - \frac{5}{5096}\sigma_5\left(\frac{n}{3}\right)$$
$$-\frac{1}{5733}\sigma_5\left(\frac{n}{4}\right) + \frac{5}{5096}\sigma_5\left(\frac{n}{6}\right) + \frac{64}{5733}\sigma_5\left(\frac{n}{8}\right) - \frac{10}{637}\sigma_5\left(\frac{n}{12}\right) + \frac{640}{637}\sigma_5\left(\frac{n}{24}\right)$$
$$-\frac{1168}{819}a_1(n) - \frac{864}{91}a_1\left(\frac{n}{2}\right) - \frac{71\,104}{819}a_1\left(\frac{n}{4}\right) - \frac{231\,424}{273}a_1\left(\frac{n}{8}\right) + \frac{7}{6}a_2(n)$$
$$+\frac{56}{3}a_2\left(\frac{n}{2}\right) - 39a_2\left(\frac{n}{3}\right) - 48a_2\left(\frac{n}{6}\right) + \frac{16}{63}a_3(n) - \frac{32}{3}a_3\left(\frac{n}{2}\right)$$
$$+\frac{5632}{63}a_3\left(\frac{n}{4}\right) + 72a_4\left(\frac{n}{3}\right) + 128a_5(n) - 384a_6(n) - 512a_7(n),$$

$$N(2^4,3^6,6^2;n) = -\frac{1}{45\,864}\sigma_5(n) + \frac{1}{45\,864}\sigma_5\left(\frac{n}{2}\right) - \frac{5}{2548}\sigma_5\left(\frac{n}{3}\right)$$
$$-\frac{1}{5733}\sigma_5\left(\frac{n}{4}\right) + \frac{5}{2548}\sigma_5\left(\frac{n}{6}\right) + \frac{64}{5733}\sigma_5\left(\frac{n}{6}\right) - \frac{10}{637}\sigma_5\left(\frac{n}{8}\right) + \frac{640}{637}\sigma_5\left(\frac{n}{12}\right)$$
$$-\frac{1790}{819}\sigma_5\left(\frac{n}{24}\right) - \frac{15\,188}{819}a_1(n) - \frac{71\,264}{273}a_1\left(\frac{n}{2}\right) - \frac{604\,160}{273}a_1\left(\frac{n}{4}\right) + \frac{5}{3}a_1\left(\frac{n}{8}\right)$$
$$+\frac{88}{3}a_2(n) - 60a_2\left(\frac{n}{2}\right) - 48a_2\left(\frac{n}{3}\right) + \frac{32}{63}a_2\left(\frac{n}{6}\right) - \frac{160}{9}a_3(n) + \frac{4864}{21}a_3\left(\frac{n}{2}\right)$$
$$+108a_4(n) + 128a_4\left(\frac{n}{3}\right) - 384a_5(n) - 1024a_6(n),$$
$$N(2^4,3^8;n) = -\frac{1}{22\,932}\sigma_5(n) + \frac{1}{22\,932}\sigma_5\left(\frac{n}{2}\right) - \frac{5}{1274}\sigma_5\left(\frac{n}{3}\right)$$
$$-\frac{1}{5733}\sigma_5\left(\frac{n}{4}\right) + \frac{5}{1274}\sigma_5\left(\frac{n}{6}\right) + \frac{64}{5733}\sigma_5\left(\frac{n}{8}\right) - \frac{10}{637}\sigma_5\left(\frac{n}{12}\right)$$
$$+\frac{640}{637}\sigma_5\left(\frac{n}{24}\right) - \frac{2488}{819}a_1(n) - \frac{8912}{273}a_1\left(\frac{n}{2}\right) - \frac{577\,792}{819}a_1\left(\frac{n}{4}\right)$$

$$-\tfrac{512\,000}{91}a_1\left(\tfrac{n}{8}\right)+2a_2\left(n\right)+40a_2\left(\tfrac{n}{2}\right)-84a_2\left(\tfrac{n}{3}\right)-48a_2\left(\tfrac{n}{6}\right)$$
$$+\tfrac{64}{63}a_3\left(n\right)-\tfrac{64}{3}a_3\left(\tfrac{n}{2}\right)+\tfrac{37\,888}{63}a_3\left(\tfrac{n}{4}\right)+144a_4\left(\tfrac{n}{3}\right)-2048a_7\left(n\right),$$

$$N(2^6,3^2,6^4;n)=-\tfrac{1}{52\,416}\sigma_5\left(n\right)+\tfrac{1}{52\,416}\sigma_5\left(\tfrac{n}{2}\right)+\tfrac{3}{5824}\sigma_5\left(\tfrac{n}{3}\right)$$
$$+\tfrac{1}{1638}\sigma_5\left(\tfrac{n}{4}\right)-\tfrac{3}{5824}\sigma_5\left(\tfrac{n}{6}\right)-\tfrac{32}{819}\sigma_5\left(\tfrac{n}{8}\right)-\tfrac{3}{182}\sigma_5\left(\tfrac{n}{12}\right)$$
$$+\tfrac{96}{91}\sigma_5\left(\tfrac{n}{24}\right)+\tfrac{319}{117}a_1\left(n\right)+\tfrac{782}{39}a_1\left(\tfrac{n}{2}\right)+\tfrac{8720}{117}a_1\left(\tfrac{n}{4}\right)$$
$$+\tfrac{12\,544}{39}a_1\left(\tfrac{n}{8}\right)-\tfrac{23}{24}a_2\left(n\right)-\tfrac{56}{3}a_2\left(\tfrac{n}{2}\right)+\tfrac{591}{8}a_2\left(\tfrac{n}{3}\right)$$
$$+264a_2\left(\tfrac{n}{6}\right)-\tfrac{16}{9}a_3\left(n\right)+32a_3\left(\tfrac{n}{2}\right)-\tfrac{320}{9}a_3\left(\tfrac{n}{4}\right)$$
$$-108a_4\left(\tfrac{n}{3}\right)+224a_5\left(n\right)+480a_6\left(n\right)+384a_7\left(n\right),$$

$$N(2^6,3^4,6^2;n)=-\tfrac{1}{26\,208}\sigma_5\left(n\right)+\tfrac{1}{26\,208}\sigma_5\left(\tfrac{n}{2}\right)+\tfrac{3}{2912}\sigma_5\left(\tfrac{n}{3}\right)$$
$$+\tfrac{1}{1638}\sigma_5\left(\tfrac{n}{4}\right)-\tfrac{3}{2912}\sigma_5\left(\tfrac{n}{6}\right)-\tfrac{32}{819}\sigma_5\left(\tfrac{n}{8}\right)-\tfrac{3}{182}\sigma_5\left(\tfrac{n}{12}\right)$$
$$+\tfrac{96}{91}\sigma_5\left(\tfrac{n}{24}\right)+\tfrac{638}{117}a_1\left(n\right)+\tfrac{1460}{39}a_1\left(\tfrac{n}{2}\right)+\tfrac{15\,376}{117}a_1\left(\tfrac{n}{4}\right)$$
$$+\tfrac{15\,872}{39}a_1\left(\tfrac{n}{8}\right)-\tfrac{23}{12}a_2\left(n\right)-\tfrac{124}{3}a_2\left(\tfrac{n}{2}\right)+\tfrac{591}{4}a_2\left(\tfrac{n}{3}\right)$$
$$+348a_2\left(\tfrac{n}{6}\right)-\tfrac{32}{9}a_3\left(n\right)+\tfrac{176}{3}a_3\left(\tfrac{n}{2}\right)-\tfrac{1024}{9}a_3\left(\tfrac{n}{4}\right)$$
$$-216a_4\left(\tfrac{n}{3}\right)+448a_5\left(n\right)+960a_6\left(n\right)+768a_7\left(n\right),$$

$$N(2^6,3^6;n)=-\tfrac{1}{13\,104}\sigma_5\left(n\right)+\tfrac{1}{13\,104}\sigma_5\left(\tfrac{n}{2}\right)+\tfrac{3}{1456}\sigma_5\left(\tfrac{n}{3}\right)+\tfrac{1}{1638}\sigma_5\left(\tfrac{n}{4}\right)$$
$$-\tfrac{3}{1456}\sigma_5\left(\tfrac{n}{6}\right)-\tfrac{32}{819}\sigma_5\left(\tfrac{n}{8}\right)-\tfrac{3}{182}\sigma_5\left(\tfrac{n}{12}\right)+\tfrac{96}{91}\sigma_5\left(\tfrac{n}{24}\right)+\tfrac{938}{117}a_1\left(n\right)$$
$$+\tfrac{2608}{39}a_1\left(\tfrac{n}{2}\right)+\tfrac{53\,440}{117}a_1\left(\tfrac{n}{4}\right)+\tfrac{95\,744}{39}a_1\left(\tfrac{n}{8}\right)-\tfrac{19}{6}a_2\left(n\right)-\tfrac{172}{3}a_2\left(\tfrac{n}{2}\right)$$
$$+\tfrac{435}{2}a_2\left(\tfrac{n}{3}\right)+348a_2\left(\tfrac{n}{6}\right)-\tfrac{44}{9}a_3\left(n\right)+64a_3\left(\tfrac{n}{2}\right)-\tfrac{2944}{9}a_3\left(\tfrac{n}{4}\right)$$
$$-324a_4\left(\tfrac{n}{3}\right)+448a_5\left(n\right)+960a_6\left(n\right)+1536a_7\left(n\right),$$

$$N(2^8,3^2,6^2;n)=-\tfrac{5}{91\,728}\sigma_5\left(n\right)+\tfrac{5}{91\,728}\sigma_5\left(\tfrac{n}{2}\right)-\tfrac{9}{20\,384}\sigma_5\left(\tfrac{n}{3}\right)$$
$$-\tfrac{10}{5733}\sigma_5\left(\tfrac{n}{4}\right)+\tfrac{9}{20\,384}\sigma_5\left(\tfrac{n}{6}\right)+\tfrac{640}{5733}\sigma_5\left(\tfrac{n}{8}\right)-\tfrac{9}{637}\sigma_5\left(\tfrac{n}{12}\right)$$
$$+\tfrac{576}{637}\sigma_5\left(\tfrac{n}{24}\right)-\tfrac{6568}{819}a_1\left(n\right)-\tfrac{27\,232}{819}a_1\left(\tfrac{n}{2}\right)+\tfrac{56\,128}{273}a_1\left(\tfrac{n}{4}\right)$$
$$+\tfrac{191\,488}{91}a_1\left(\tfrac{n}{8}\right)+\tfrac{19}{6}a_2\left(n\right)+64a_2\left(\tfrac{n}{2}\right)-\tfrac{867}{4}a_2\left(\tfrac{n}{3}\right)-384a_2\left(\tfrac{n}{6}\right)$$
$$+\tfrac{304}{63}a_3\left(n\right)-\tfrac{704}{9}a_3\left(\tfrac{n}{2}\right)+\tfrac{256}{21}a_3\left(\tfrac{n}{4}\right)+324a_4\left(\tfrac{n}{3}\right)-384a_5\left(n\right)$$
$$-1920a_6\left(n\right)-512a_7\left(n\right),$$

$$N(2^8,3^4;n)=-\tfrac{5}{45\,864}\sigma_5\left(n\right)+\tfrac{5}{45\,864}\sigma_5\left(\tfrac{n}{2}\right)-\tfrac{9}{10\,192}\sigma_5\left(\tfrac{n}{3}\right)$$
$$-\tfrac{10}{5733}\sigma_5\left(\tfrac{n}{4}\right)+\tfrac{9}{10\,192}\sigma_5\left(\tfrac{n}{6}\right)+\tfrac{640}{5733}\sigma_5\left(\tfrac{n}{8}\right)-\tfrac{9}{637}\sigma_5\left(\tfrac{n}{12}\right)$$
$$+\tfrac{576}{637}\sigma_5\left(\tfrac{n}{24}\right)-\tfrac{13\,136}{819}a_1\left(n\right)-\tfrac{80\,672}{819}a_1\left(\tfrac{n}{2}\right)+\tfrac{28\,416}{91}a_1\left(\tfrac{n}{4}\right)$$
$$+\tfrac{1040\,384}{273}a_1\left(\tfrac{n}{8}\right)+\tfrac{19}{3}a_2\left(n\right)+\tfrac{368}{3}a_2\left(\tfrac{n}{2}\right)-\tfrac{867}{2}a_2\left(\tfrac{n}{3}\right)$$
$$-984a_2\left(\tfrac{n}{6}\right)+\tfrac{608}{63}a_3\left(n\right)-\tfrac{1216}{9}a_3\left(\tfrac{n}{2}\right)-\tfrac{5120}{21}a_3\left(\tfrac{n}{4}\right)$$
$$+648a_4\left(\tfrac{n}{3}\right)-768a_5\left(n\right)-3840a_6\left(n\right)-1024a_7\left(n\right),$$

$$N(2^{10},3^2;n)=-\tfrac{61}{366\,912}\sigma_5\left(n\right)+\tfrac{61}{366\,912}\sigma_5\left(\tfrac{n}{2}\right)+\tfrac{27}{40\,768}\sigma_5\left(\tfrac{n}{3}\right)$$
$$+\tfrac{61}{11\,466}\sigma_5\left(\tfrac{n}{4}\right)-\tfrac{27}{40\,768}\sigma_5\left(\tfrac{n}{6}\right)-\tfrac{1952}{5733}\sigma_5\left(\tfrac{n}{8}\right)-\tfrac{27}{1274}\sigma_5\left(\tfrac{n}{12}\right)$$
$$+\tfrac{864}{637}\sigma_5\left(\tfrac{n}{24}\right)+\tfrac{18\,497}{819}a_1\left(n\right)+\tfrac{86\,894}{819}a_1\left(\tfrac{n}{2}\right)-\tfrac{307\,472}{273}a_1\left(\tfrac{n}{4}\right)$$
$$-\tfrac{2190\,080}{273}a_1\left(\tfrac{n}{8}\right)-\tfrac{95}{8}a_2\left(n\right)-\tfrac{488}{3}a_2\left(\tfrac{n}{2}\right)+\tfrac{4881}{8}a_2\left(\tfrac{n}{3}\right)$$
$$+2424a_2\left(\tfrac{n}{6}\right)-\tfrac{680}{63}a_3\left(n\right)+\tfrac{2176}{9}a_3\left(\tfrac{n}{2}\right)+\tfrac{18\,752}{21}a_3\left(\tfrac{n}{4}\right)$$
$$-972a_4\left(\tfrac{n}{3}\right)-96a_5\left(n\right)+7584a_6\left(n\right)+640a_7\left(n\right),$$

$$N(1,2,3,6^9;n)=-\tfrac{1}{733\,824}\sigma_5\left(n\right)+\tfrac{1}{733\,824}\sigma_5\left(\tfrac{n}{2}\right)-\tfrac{121}{244\,608}\sigma_5\left(\tfrac{n}{3}\right)$$
$$-\tfrac{1}{22\,932}\sigma_5\left(\tfrac{n}{4}\right)+\tfrac{121}{244\,608}\sigma_5\left(\tfrac{n}{6}\right)+\tfrac{16}{5733}\sigma_5\left(\tfrac{n}{8}\right)-\tfrac{121}{7644}\sigma_5\left(\tfrac{n}{12}\right)$$
$$+\tfrac{1936}{1911}\sigma_5\left(\tfrac{n}{24}\right)-\tfrac{67}{546}a_1\left(n\right)+\tfrac{757}{819}a_1\left(\tfrac{n}{2}\right)+\tfrac{113\,992}{819}a_1\left(\tfrac{n}{4}\right)+\tfrac{70\,016}{91}a_1\left(\tfrac{n}{8}\right)$$
$$+\tfrac{5}{16}a_2\left(n\right)-\tfrac{57}{16}a_2\left(\tfrac{n}{3}\right)+72a_2\left(\tfrac{n}{6}\right)-\tfrac{4}{21}a_3\left(n\right)-\tfrac{64}{9}a_3\left(\tfrac{n}{2}\right)$$
$$-\tfrac{6880}{63}a_3\left(\tfrac{n}{4}\right)+2a_4\left(n\right)+6a_4\left(\tfrac{n}{3}\right)+176a_5\left(n\right)-592a_6\left(n\right)+192a_7\left(n\right),$$

$$N(1,2,3^3,6^7;n)=-\tfrac{1}{366\,912}\sigma_5\left(n\right)+\tfrac{1}{366\,912}\sigma_5\left(\tfrac{n}{2}\right)-\tfrac{121}{122\,304}\sigma_5\left(\tfrac{n}{3}\right)-\tfrac{1}{22\,932}\sigma_5\left(\tfrac{n}{4}\right)$$
$$+\tfrac{121}{122\,304}\sigma_5\left(\tfrac{n}{6}\right)+\tfrac{16}{5733}\sigma_5\left(\tfrac{n}{8}\right)-\tfrac{121}{7644}\sigma_5\left(\tfrac{n}{12}\right)+\tfrac{1936}{1911}\sigma_5\left(\tfrac{n}{24}\right)-\tfrac{493}{1638}a_1\left(n\right)$$
$$+\tfrac{331}{819}a_1\left(\tfrac{n}{2}\right)+\tfrac{36\,784}{273}a_1\left(\tfrac{n}{4}\right)+\tfrac{58\,368}{91}a_1\left(\tfrac{n}{8}\right)+\tfrac{11}{24}a_2\left(n\right)+2a_2\left(\tfrac{n}{2}\right)-\tfrac{69}{8}a_2\left(\tfrac{n}{3}\right)$$
$$+54a_2\left(\tfrac{n}{6}\right)-\tfrac{10}{63}a_3\left(n\right)-\tfrac{88}{9}a_3\left(\tfrac{n}{2}\right)-\tfrac{640}{7}a_3\left(\tfrac{n}{4}\right)+2a_4\left(n\right)+18a_4\left(\tfrac{n}{3}\right)+192a_5\left(n\right)$$
$$-640a_6\left(n\right)+128a_7\left(n\right),$$

$$N(1,2,3^5,6^5;n)=-\tfrac{1}{183\,456}\sigma_5\left(n\right)+\tfrac{1}{183\,456}\sigma_5\left(\tfrac{n}{2}\right)-\tfrac{121}{61\,152}\sigma_5\left(\tfrac{n}{3}\right)$$

$$-\tfrac{1}{22\,932}\sigma_5\left(\tfrac{n}{4}\right)+\tfrac{121}{61\,152}\sigma_5\left(\tfrac{n}{6}\right)+\tfrac{16}{5733}\sigma_5\left(\tfrac{n}{8}\right)-\tfrac{121}{7644}\sigma_5\left(\tfrac{n}{12}\right)$$
$$+\tfrac{1936}{1911}\sigma_5\left(\tfrac{n}{24}\right)-\tfrac{134}{273}a_1(n)-\tfrac{794}{819}a_1\left(\tfrac{n}{2}\right)+\tfrac{87\,784}{819}a_1\left(\tfrac{n}{4}\right)$$
$$+\tfrac{81\,920}{273}a_1\left(\tfrac{n}{8}\right)+\tfrac{7}{12}a_2(n)+\tfrac{14}{3}a_2\left(\tfrac{n}{2}\right)-\tfrac{57}{4}a_2\left(\tfrac{n}{3}\right)+54a_2\left(\tfrac{n}{6}\right)$$
$$-\tfrac{2}{21}a_3(n)-\tfrac{112}{9}a_3\left(\tfrac{n}{2}\right)-\tfrac{3520}{63}a_3\left(\tfrac{n}{4}\right)+2a_4(n)+30a_4\left(\tfrac{n}{3}\right)$$
$$+224a_5(n)-672a_6(n),$$

$$N(1,2,3^7,6^3;n)=-\tfrac{1}{91\,728}\sigma_5(n)+\tfrac{1}{91\,728}\sigma_5\left(\tfrac{n}{2}\right)-\tfrac{121}{30\,576}\sigma_5\left(\tfrac{n}{3}\right)$$
$$-\tfrac{1}{22\,932}\sigma_5\left(\tfrac{n}{4}\right)+\tfrac{121}{30\,576}\sigma_5\left(\tfrac{n}{6}\right)+\tfrac{16}{5733}\sigma_5\left(\tfrac{n}{8}\right)-\tfrac{121}{7644}\sigma_5\left(\tfrac{n}{12}\right)$$
$$+\tfrac{1936}{1911}\sigma_5\left(\tfrac{n}{24}\right)-\tfrac{1153}{1638}a_1(n)-\tfrac{863}{273}a_1\left(\tfrac{n}{2}\right)+\tfrac{28\,088}{819}a_1\left(\tfrac{n}{4}\right)$$
$$-\tfrac{127\,744}{273}a_1\left(\tfrac{n}{8}\right)+\tfrac{2}{3}a_2(n)+\tfrac{26}{3}a_2\left(\tfrac{n}{2}\right)-21a_2\left(\tfrac{n}{3}\right)+66a_2\left(\tfrac{n}{6}\right)$$
$$+\tfrac{2}{63}a_3(n)-16a_3\left(\tfrac{n}{2}\right)+\tfrac{1408}{63}a_3\left(\tfrac{n}{4}\right)+2a_4(n)$$
$$+42a_4\left(\tfrac{n}{3}\right)+256a_5(n)-640a_6(n)-256a_7(n),$$
$$N(1,2,3^9,6;n)=-\tfrac{1}{45\,864}\sigma_5(n)+\tfrac{1}{45\,864}\sigma_5\left(\tfrac{n}{2}\right)-\tfrac{121}{15\,288}\sigma_5\left(\tfrac{n}{3}\right)$$
$$-\tfrac{1}{22\,932}\sigma_5\left(\tfrac{n}{4}\right)+\tfrac{121}{15\,288}\sigma_5\left(\tfrac{n}{6}\right)+\tfrac{16}{5733}\sigma_5\left(\tfrac{n}{8}\right)-\tfrac{121}{7644}\sigma_5\left(\tfrac{n}{12}\right)$$
$$+\tfrac{1936}{1911}\sigma_5\left(\tfrac{n}{24}\right)-\tfrac{263}{273}a_1(n)-\tfrac{4996}{819}a_1\left(\tfrac{n}{2}\right)-\tfrac{105\,136}{819}a_1\left(\tfrac{n}{4}\right)$$
$$-\tfrac{570\,368}{273}a_1\left(\tfrac{n}{8}\right)+\tfrac{2}{3}a_2(n)+\tfrac{46}{3}a_2\left(\tfrac{n}{2}\right)-30a_2\left(\tfrac{n}{3}\right)$$
$$+78a_1\left(\tfrac{n}{6}\right)+\tfrac{2}{7}a_3(n)-\tfrac{200}{9}a_3\left(\tfrac{n}{2}\right)+\tfrac{12\,160}{63}a_3\left(\tfrac{n}{4}\right)+2a_4(n)$$
$$+54a_4\left(\tfrac{n}{3}\right)+256a_5(n)-512a_6(n)-768a_7(n),$$
$$N(1,2^3,3,6^7;n)=-\tfrac{5}{733\,824}\sigma_5(n)+\tfrac{5}{733\,824}\sigma_5\left(\tfrac{n}{2}\right)+\tfrac{41}{81\,536}\sigma_5\left(\tfrac{n}{3}\right)$$
$$+\tfrac{5}{22\,932}\sigma_5\left(\tfrac{n}{4}\right)-\tfrac{41}{81\,536}\sigma_5\left(\tfrac{n}{6}\right)-\tfrac{80}{5733}\sigma_5\left(\tfrac{n}{8}\right)-\tfrac{41}{2548}\sigma_5\left(\tfrac{n}{12}\right)$$
$$+\tfrac{656}{637}\sigma_5\left(\tfrac{n}{24}\right)+\tfrac{131}{182}a_1(n)+\tfrac{6697}{819}a_1\left(\tfrac{n}{2}\right)+\tfrac{19\,016}{91}a_1\left(\tfrac{n}{4}\right)$$
$$+\tfrac{370\,816}{273}a_1\left(\tfrac{n}{8}\right)+\tfrac{11}{48}a_2(n)-\tfrac{20}{3}a_2\left(\tfrac{n}{2}\right)+\tfrac{315}{16}a_2\left(\tfrac{n}{3}\right)+84a_2\left(\tfrac{n}{6}\right)$$
$$-\tfrac{20}{21}a_3(n)+\tfrac{32}{9}a_3\left(\tfrac{n}{2}\right)-\tfrac{3616}{21}a_3\left(\tfrac{n}{4}\right)+2a_4(n)-18a_4\left(\tfrac{n}{3}\right)$$
$$+240a_5(n)-528a_6(n)+448a_7(n),$$
$$N(1,2^3,3^3,6^5;n)=-\tfrac{5}{366\,912}\sigma_5(n)+\tfrac{5}{366\,912}\sigma_5\left(\tfrac{n}{2}\right)+\tfrac{41}{40\,768}\sigma_5\left(\tfrac{n}{3}\right)$$
$$+\tfrac{5}{22\,932}\sigma_5\left(\tfrac{n}{4}\right)-\tfrac{41}{40\,768}\sigma_5\left(\tfrac{n}{6}\right)-\tfrac{80}{5733}\sigma_5\left(\tfrac{n}{8}\right)-\tfrac{41}{2548}\sigma_5\left(\tfrac{n}{12}\right)$$
$$+\tfrac{656}{637}\sigma_5\left(\tfrac{n}{24}\right)+\tfrac{2813}{1638}a_1(n)+\tfrac{10\,937}{819}a_1\left(\tfrac{n}{2}\right)+\tfrac{192\,256}{819}a_1\left(\tfrac{n}{4}\right)$$
$$+\tfrac{429\,056}{273}a_1\left(\tfrac{n}{8}\right)-\tfrac{1}{24}a_2(n)-\tfrac{46}{3}a_2\left(\tfrac{n}{2}\right)+\tfrac{375}{8}a_2\left(\tfrac{n}{3}\right)+162a_2\left(\tfrac{n}{6}\right)$$
$$-\tfrac{106}{63}a_3(n)+\tfrac{136}{9}a_3\left(\tfrac{n}{2}\right)-\tfrac{13\,312}{63}a_3\left(\tfrac{n}{4}\right)+2a_4(n)-54a_4\left(\tfrac{n}{3}\right)$$
$$+320a_5(n)-384a_6(n)+640a_7(n),$$
$$N(1,2^3,3^5,6^3;n)=-\tfrac{5}{183\,456}\sigma_5(n)+\tfrac{5}{183\,456}\sigma_5\left(\tfrac{n}{2}\right)+\tfrac{41}{20\,384}\sigma_5\left(\tfrac{n}{3}\right)$$
$$+\tfrac{5}{22\,932}\sigma_5\left(\tfrac{n}{4}\right)-\tfrac{41}{20\,384}\sigma_5\left(\tfrac{n}{6}\right)-\tfrac{80}{5733}\sigma_5\left(\tfrac{n}{8}\right)-\tfrac{41}{2548}\sigma_5\left(\tfrac{n}{12}\right)$$
$$+\tfrac{656}{637}\sigma_5\left(\tfrac{n}{24}\right)+\tfrac{2176}{819}a_1(n)+\tfrac{17\,506}{819}a_1\left(\tfrac{n}{2}\right)+\tfrac{274\,520}{819}a_1\left(\tfrac{n}{4}\right)$$
$$+\tfrac{220\,672}{91}a_1\left(\tfrac{n}{8}\right)-\tfrac{5}{12}a_2(n)-26a_2\left(\tfrac{n}{2}\right)+\tfrac{291}{4}a_2\left(\tfrac{n}{3}\right)+186a_2\left(\tfrac{n}{6}\right)$$
$$-\tfrac{142}{63}a_3(n)+\tfrac{224}{9}a_3\left(\tfrac{n}{2}\right)-\tfrac{19\,136}{63}a_3\left(\tfrac{n}{4}\right)+2a_4(n)-90a_4\left(\tfrac{n}{3}\right)$$
$$+352a_5(n)-288a_6(n)+1024a_7(n),$$

$$N(1,2^3,3^7,6;n)=-\tfrac{5}{91\,728}\sigma_5(n)+\tfrac{5}{91\,728}\sigma_5\left(\tfrac{n}{2}\right)+\tfrac{41}{10\,192}\sigma_5\left(\tfrac{n}{3}\right)$$
$$+\tfrac{5}{22\,932}\sigma_5\left(\tfrac{n}{4}\right)-\tfrac{41}{10\,192}\sigma_5\left(\tfrac{n}{6}\right)-\tfrac{80}{5733}\sigma_5\left(\tfrac{n}{8}\right)-\tfrac{41}{2548}\sigma_5\left(\tfrac{n}{12}\right)$$
$$+\tfrac{656}{637}\sigma_5\left(\tfrac{n}{24}\right)+\tfrac{5701}{1638}a_1(n)+\tfrac{2869}{91}a_1\left(\tfrac{n}{2}\right)+\tfrac{488\,552}{819}a_1\left(\tfrac{n}{4}\right)$$
$$+\tfrac{430\,336}{91}a_1\left(\tfrac{n}{8}\right)-a_2(n)-38a_2\left(\tfrac{n}{2}\right)+96a_2\left(\tfrac{n}{3}\right)+150a_2\left(\tfrac{n}{6}\right)$$
$$-\tfrac{158}{63}a_3(n)+32a_3\left(\tfrac{n}{2}\right)-\tfrac{33\,920}{63}a_3\left(\tfrac{n}{4}\right)+2a_4(n)-126a_4\left(\tfrac{n}{3}\right)$$
$$+256a_5(n)-384a_6(n)+1792a_7(n),$$
$$N(1,2^5,3,6^5;n)=-\tfrac{1}{56\,448}\sigma_5(n)+\tfrac{1}{56\,448}\sigma_5\left(\tfrac{n}{2}\right)-\tfrac{3}{6272}\sigma_5\left(\tfrac{n}{3}\right)$$
$$-\tfrac{1}{1764}\sigma_5\left(\tfrac{n}{4}\right)+\tfrac{3}{6272}\sigma_5\left(\tfrac{n}{6}\right)+\tfrac{16}{441}\sigma_5\left(\tfrac{n}{8}\right)-\tfrac{3}{196}\sigma_5\left(\tfrac{n}{12}\right)+\tfrac{48}{49}\sigma_5\left(\tfrac{n}{24}\right)$$
$$-\tfrac{89}{126}a_1(n)-\tfrac{251}{63}a_1\left(\tfrac{n}{2}\right)+\tfrac{6920}{63}a_1\left(\tfrac{n}{4}\right)+\tfrac{9344}{21}a_1\left(\tfrac{n}{8}\right)+\tfrac{67}{48}a_2(n)$$
$$+\tfrac{32}{3}a_2\left(\tfrac{n}{2}\right)-\tfrac{309}{16}a_2\left(\tfrac{n}{3}\right)+72a_2\left(\tfrac{n}{6}\right)-\tfrac{44}{63}a_3(n)-\tfrac{64}{9}a_3\left(\tfrac{n}{2}\right)$$
$$-\tfrac{5216}{63}a_3\left(\tfrac{n}{4}\right)+2a_4(n)+54a_4\left(\tfrac{n}{3}\right)+368a_5(n)-912a_6(n)-64a_7(n),$$
$$N(1,2^5,3^3,6^3;n)=-\tfrac{1}{28\,224}\sigma_5(n)+\tfrac{1}{28\,224}\sigma_5\left(\tfrac{n}{2}\right)-\tfrac{3}{3136}\sigma_5\left(\tfrac{n}{3}\right)$$
$$-\tfrac{1}{1764}\sigma_5\left(\tfrac{n}{4}\right)+\tfrac{3}{3136}\sigma_5\left(\tfrac{n}{6}\right)+\tfrac{16}{441}\sigma_5\left(\tfrac{n}{8}\right)-\tfrac{3}{196}\sigma_5\left(\tfrac{n}{12}\right)$$
$$+\tfrac{48}{49}\sigma_5\left(\tfrac{n}{24}\right)-\tfrac{409}{126}a_1(n)-\tfrac{347}{21}a_1\left(\tfrac{n}{2}\right)+\tfrac{7088}{63}a_1\left(\tfrac{n}{4}\right)+\tfrac{10\,240}{21}a_1\left(\tfrac{n}{8}\right)$$

$$
\begin{aligned}
&+\frac{21}{8}a_2(n)+\frac{94}{3}a_2\left(\frac{n}{2}\right)-\frac{705}{8}a_2\left(\frac{n}{3}\right)-114a_2\left(\frac{n}{6}\right)+\frac{38}{63}a_3(n)\\
&-\frac{104}{3}a_3\left(\frac{n}{2}\right)-\frac{3200}{63}a_3\left(\frac{n}{4}\right)+2a_4(n)+162a_4\left(\frac{n}{3}\right)+320a_5(n)\\
&-1536a_6(n)-384a_7(n),
\end{aligned}
$$

$$
\begin{aligned}
&N(1,2^5,3^5,6;n)=-\frac{1}{14\,112}\sigma_5(n)+\frac{1}{14\,112}\sigma_5\left(\frac{n}{2}\right)-\frac{3}{1568}\sigma_5\left(\frac{n}{3}\right)\\
&-\frac{1}{1764}\sigma_5\left(\frac{n}{4}\right)+\frac{3}{1568}\sigma_5\left(\frac{n}{6}\right)+\frac{16}{441}\sigma_5\left(\frac{n}{8}\right)-\frac{3}{196}\sigma_5\left(\frac{n}{12}\right)\\
&+\frac{48}{49}\sigma_5\left(\frac{n}{24}\right)-\frac{374}{63}a_1(n)-\frac{2642}{63}a_1\left(\frac{n}{2}\right)-\frac{1576}{21}a_1\left(\frac{n}{4}\right)\\
&-\frac{11\,264}{21}a_1\left(\frac{n}{8}\right)+\frac{43}{12}a_2(n)+\frac{166}{3}a_2\left(\frac{n}{2}\right)-\frac{645}{4}a_2\left(\frac{n}{3}\right)\\
&-210a_2\left(\frac{n}{6}\right)+\frac{146}{63}a_3(n)-\frac{496}{9}a_3\left(\frac{n}{2}\right)+\frac{1472}{21}a_3\left(\frac{n}{4}\right)\\
&+2a_4(n)+270a_4\left(\frac{n}{3}\right)+96a_5(n)-1824a_6(n)-1024a_7(n),
\end{aligned}
$$

$$
\begin{aligned}
&N(1,2^7,3,6^3;n)=-\frac{41}{733\,824}\sigma_5(n)+\frac{41}{733\,824}\sigma_5\left(\frac{n}{2}\right)+\frac{45}{81\,536}\sigma_5\left(\frac{n}{3}\right)\\
&+\frac{41}{22\,932}\sigma_5\left(\frac{n}{4}\right)-\frac{45}{81\,536}\sigma_5\left(\frac{n}{6}\right)-\frac{656}{5733}\sigma_5\left(\frac{n}{8}\right)-\frac{45}{2548}\sigma_5\left(\frac{n}{12}\right)\\
&+\frac{720}{637}\sigma_5\left(\frac{n}{24}\right)+\frac{7775}{1638}a_1(n)+\frac{27\,397}{819}a_1\left(\frac{n}{2}\right)+\frac{57\,784}{273}a_1\left(\frac{n}{4}\right)\\
&+\frac{170\,624}{273}a_1\left(\frac{n}{8}\right)-\frac{25}{48}a_2(n)-\frac{100}{3}a_2\left(\frac{n}{2}\right)+\frac{2055}{16}a_2\left(\frac{n}{3}\right)+420a_2\left(\frac{n}{6}\right)\\
&-\frac{268}{63}a_3(n)+\frac{416}{9}a_3\left(\frac{n}{2}\right)-\frac{3488}{21}a_3\left(\frac{n}{4}\right)+2a_4(n)-162a_4\left(\frac{n}{3}\right)\\
&+816a_5(n)+48a_6(n)+704a_7(n),
\end{aligned}
$$

$$
\begin{aligned}
&N(1,2^7,3^3,6;n)=-\frac{41}{366\,912}\sigma_5(n)+\frac{41}{366\,912}\sigma_5\left(\frac{n}{2}\right)+\frac{45}{40\,768}\sigma_5\left(\frac{n}{3}\right)\\
&+\frac{41}{22\,932}\sigma_5\left(\frac{n}{4}\right)-\frac{45}{40\,768}\sigma_5\left(\frac{n}{6}\right)-\frac{656}{5733}\sigma_5\left(\frac{n}{8}\right)-\frac{45}{2548}\sigma_5\left(\frac{n}{12}\right)\\
&+\frac{720}{637}\sigma_5\left(\frac{n}{24}\right)+\frac{6791}{546}a_1(n)+\frac{19\,387}{273}a_1\left(\frac{n}{2}\right)-\frac{2336}{91}a_1\left(\frac{n}{4}\right)\\
&-\frac{376\,832}{273}a_1\left(\frac{n}{8}\right)-\frac{101}{24}a_2(n)-\frac{278}{3}a_2\left(\frac{n}{2}\right)+\frac{2691}{8}a_2\left(\frac{n}{3}\right)\\
&+954a_2\left(\frac{n}{6}\right)-\frac{58}{7}a_3(n)+\frac{376}{3}a_3\left(\frac{n}{2}\right)-\frac{1024}{21}a_3\left(\frac{n}{4}\right)+2a_4(n)\\
&-486a_4\left(\frac{n}{3}\right)+960a_5(n)+2304a_6(n)+1152a_7(n),
\end{aligned}
$$

$$
\begin{aligned}
&N(1,2^9,3,6;n)=-\frac{121}{733\,824}\sigma_5(n)+\frac{121}{733\,824}\sigma_5\left(\frac{n}{2}\right)-\frac{27}{81\,536}\sigma_5\left(\frac{n}{3}\right)\\
&-\frac{121}{22\,932}\sigma_5\left(\frac{n}{4}\right)+\frac{27}{81\,536}\sigma_5\left(\frac{n}{6}\right)+\frac{1936}{5733}\sigma_5\left(\frac{n}{8}\right)-\frac{27}{2548}\sigma_5\left(\frac{n}{12}\right)\\
&+\frac{432}{637}\sigma_5\left(\frac{n}{24}\right)-\frac{18\,497}{1638}a_1(n)-\frac{33\,619}{819}a_1\left(\frac{n}{2}\right)+\frac{163\,672}{273}a_1\left(\frac{n}{4}\right)\\
&+\frac{1187\,968}{273}a_1\left(\frac{n}{8}\right)+\frac{93}{16}a_2(n)+\frac{256}{3}a_2\left(\frac{n}{2}\right)-\frac{4881}{16}a_2\left(\frac{n}{3}\right)\\
&-888a_2\left(\frac{n}{6}\right)+\frac{340}{63}a_3(n)-\frac{1088}{9}a_3\left(\frac{n}{2}\right)-\frac{6304}{21}a_3\left(\frac{n}{4}\right)\\
&+2a_4(n)+486a_4\left(\frac{n}{3}\right)+48a_5(n)-3792a_6(n)-320a_7(n),
\end{aligned}
$$

$$
\begin{aligned}
&N(1^2,6^{10};n)=-\frac{1}{366\,912}\sigma_5(n)+\frac{1}{366\,912}\sigma_5\left(\frac{n}{2}\right)+\frac{61}{122\,304}\sigma_5\left(\frac{n}{3}\right)\\
&+\frac{1}{11\,466}\sigma_5\left(\frac{n}{4}\right)-\frac{61}{122\,304}\sigma_5\left(\frac{n}{6}\right)-\frac{32}{5733}\sigma_5\left(\frac{n}{8}\right)-\frac{61}{3822}\sigma_5\left(\frac{n}{12}\right)\\
&+\frac{1952}{1911}\sigma_5\left(\frac{n}{24}\right)-\frac{67}{273}a_1(n)+\frac{3698}{819}a_1\left(\frac{n}{2}\right)+\frac{261\,232}{819}a_1\left(\frac{n}{4}\right)\\
&+\frac{511\,744}{273}a_1\left(\frac{n}{8}\right)+\frac{5}{8}a_2(n)-\frac{8}{3}a_2\left(\frac{n}{2}\right)-\frac{51}{8}a_2\left(\frac{n}{3}\right)+120a_2\left(\frac{n}{6}\right)\\
&-\frac{8}{21}a_3(n)-\frac{128}{9}a_3\left(\frac{n}{2}\right)-\frac{15\,808}{63}a_3\left(\frac{n}{4}\right)+4a_4(n)+352a_5(n)\\
&-1184a_6(n)+384a_7(n),
\end{aligned}
$$

$$
\begin{aligned}
&N(1^2,3^2,6^8;n)=-\frac{1}{183\,456}\sigma_5(n)+\frac{1}{183\,456}\sigma_5\left(\frac{n}{2}\right)+\frac{61}{61\,152}\sigma_5\left(\frac{n}{3}\right)\\
&+\frac{1}{11\,466}\sigma_5\left(\frac{n}{4}\right)-\frac{61}{61\,152}\sigma_5\left(\frac{n}{6}\right)-\frac{32}{5733}\sigma_5\left(\frac{n}{8}\right)-\frac{61}{3822}\sigma_5\left(\frac{n}{12}\right)\\
&+\frac{1952}{1911}\sigma_5\left(\frac{n}{24}\right)+\frac{16}{91}a_1(n)+\frac{1616}{273}a_1\left(\frac{n}{2}\right)+\frac{28\,864}{91}a_1\left(\frac{n}{4}\right)\\
&+\frac{162\,816}{91}a_1\left(\frac{n}{8}\right)+\frac{7}{12}a_2(n)-4a_2\left(\frac{n}{2}\right)+\frac{21}{4}a_2\left(\frac{n}{3}\right)+156a_2\left(\frac{n}{6}\right)\\
&-\frac{16}{21}a_3(n)-\frac{32}{3}a_3\left(\frac{n}{2}\right)-\frac{5120}{21}a_3\left(\frac{n}{4}\right)+4a_4(n)-12a_4\left(\frac{n}{3}\right)\\
&+384a_5(n)-1152a_6(n)+512a_7(n),
\end{aligned}
$$

$$
\begin{aligned}
&N(1^2,3^4,6^6;n)=-\frac{1}{91\,728}\sigma_5(n)+\frac{1}{91\,728}\sigma_5\left(\frac{n}{2}\right)+\frac{61}{30\,576}\sigma_5\left(\frac{n}{3}\right)\\
&+\frac{1}{11\,466}\sigma_5\left(\frac{n}{4}\right)-\frac{61}{30\,576}\sigma_5\left(\frac{n}{6}\right)-\frac{32}{5733}\sigma_5\left(\frac{n}{8}\right)-\frac{61}{3822}\sigma_5\left(\frac{n}{12}\right)\\
&+\frac{1952}{1911}\sigma_5\left(\frac{n}{24}\right)+\frac{470}{819}a_1(n)+\frac{6784}{819}a_1\left(\frac{n}{2}\right)+\frac{256\,864}{819}a_1\left(\frac{n}{4}\right)\\
&+\frac{441\,856}{273}a_1\left(\frac{n}{8}\right)+\frac{1}{2}a_2(n)-\frac{20}{3}a_2\left(\frac{n}{2}\right)+\frac{33}{2}a_2\left(\frac{n}{3}\right)+180a_2\left(\frac{n}{6}\right)\\
&-\frac{68}{63}a_3(n)-\frac{64}{9}a_3\left(\frac{n}{2}\right)-\frac{14\,464}{63}a_3\left(\frac{n}{4}\right)+4a_4(n)-24a_4\left(\frac{n}{3}\right)\\
&+448a_5(n)-1088a_6(n)+512a_7(n),
\end{aligned}
$$

$$
\begin{aligned}
&N(1^2,3^6,6^4;n)=-\frac{1}{45\,864}\sigma_5(n)+\frac{1}{45\,864}\sigma_5\left(\frac{n}{2}\right)+\frac{61}{15\,288}\sigma_5\left(\frac{n}{3}\right)\\
&+\frac{1}{11\,466}\sigma_5\left(\frac{n}{4}\right)-\frac{61}{15\,288}\sigma_5\left(\frac{n}{6}\right)-\frac{32}{5733}\sigma_5\left(\frac{n}{8}\right)-\frac{61}{3822}\sigma_5\left(\frac{n}{12}\right)\\
&+\frac{1952}{1911}\sigma_5\left(\frac{n}{24}\right)+\frac{758}{819}a_1(n)+\frac{3188}{273}a_1\left(\frac{n}{2}\right)+\frac{262\,688}{819}a_1\left(\frac{n}{4}\right)\\
&+\frac{395\,264}{273}a_1\left(\frac{n}{8}\right)+\frac{1}{3}a_2(n)-\frac{28}{3}a_2\left(\frac{n}{2}\right)+27a_2\left(\frac{n}{3}\right)
\end{aligned}
$$

$$
\begin{aligned}
&+204a_2\left(\tfrac{n}{6}\right)-\tfrac{80}{63}a_3(n)-\tfrac{16}{3}a_3\left(\tfrac{n}{2}\right)-\tfrac{13\,568}{63}a_3\left(\tfrac{n}{4}\right)+4a_4(n)\\
&-36a_4\left(\tfrac{n}{3}\right)+512a_5(n)-1024a_6(n)+512a_7(n),\\
&N(1^2,3^8,6^2;n)=-\tfrac{1}{22\,932}\sigma_5(n)+\tfrac{1}{22\,932}\sigma_5\left(\tfrac{n}{2}\right)+\tfrac{61}{7644}\sigma_5\left(\tfrac{n}{3}\right)\\
&+\tfrac{1}{11\,466}\sigma_5\left(\tfrac{n}{4}\right)-\tfrac{61}{7644}\sigma_5\left(\tfrac{n}{6}\right)-\tfrac{32}{5733}\sigma_5\left(\tfrac{n}{8}\right)-\tfrac{61}{3822}\sigma_5\left(\tfrac{n}{12}\right)\\
&+\tfrac{1952}{1911}\sigma_5\left(\tfrac{n}{24}\right)+\tfrac{970}{819}a_1(n)+\tfrac{13\,304}{819}a_1\left(\tfrac{n}{2}\right)+\tfrac{297\,632}{819}a_1\left(\tfrac{n}{4}\right)\\
&+\tfrac{395\,264}{273}a_1\left(\tfrac{n}{8}\right)-\tfrac{28}{3}a_2\left(\tfrac{n}{2}\right)+24a_2\left(\tfrac{n}{3}\right)+204a_2\left(\tfrac{n}{6}\right)\\
&-\tfrac{76}{63}a_3(n)-\tfrac{80}{9}a_3\left(\tfrac{n}{2}\right)-\tfrac{13\,568}{63}a_3\left(\tfrac{n}{4}\right)+4a_4(n)-36a_4\left(\tfrac{n}{3}\right)\\
&+512a_5(n)-1024a_6(n)+512a_7(n),\\
&N(1^2,3^{10};n)=-\tfrac{1}{11\,466}\sigma_5(n)+\tfrac{1}{5733}\sigma_5\left(\tfrac{n}{2}\right)+\tfrac{61}{3822}\sigma_5\left(\tfrac{n}{3}\right)
\end{aligned}
$$

$$
\begin{aligned}
&-\tfrac{32}{5733}\sigma_5\left(\tfrac{n}{4}\right)-\tfrac{61}{1911}\sigma_5\left(\tfrac{n}{6}\right)+\tfrac{1952}{1911}\sigma_5\left(\tfrac{n}{12}\right)+\tfrac{1576}{819}a_1(n)\\
&+\tfrac{2848}{117}a_1\left(\tfrac{n}{2}\right)+\tfrac{23\,040}{91}a_1\left(\tfrac{n}{4}\right)+\tfrac{128}{63}a_3(n)-\tfrac{1024}{63}a_3\left(\tfrac{n}{2}\right),\\
&N(1^2,2^2,6^8;n)=-\tfrac{1}{183\,456}\sigma_5(n)+\tfrac{1}{183\,456}\sigma_5\left(\tfrac{n}{2}\right)\\
&-\tfrac{5}{10\,192}\sigma_5\left(\tfrac{n}{3}\right)-\tfrac{1}{5733}\sigma_5\left(\tfrac{n}{4}\right)+\tfrac{5}{10\,192}\sigma_5\left(\tfrac{n}{6}\right)+\tfrac{64}{5733}\sigma_5\left(\tfrac{n}{8}\right)\\
&-\tfrac{10}{637}\sigma_5\left(\tfrac{n}{12}\right)+\tfrac{640}{637}\sigma_5\left(\tfrac{n}{24}\right)+\tfrac{16}{91}a_1(n)+\tfrac{1936}{819}a_1\left(\tfrac{n}{2}\right)\\
&+\tfrac{220\,096}{819}a_1\left(\tfrac{n}{4}\right)+\tfrac{140\,288}{91}a_1\left(\tfrac{n}{8}\right)+\tfrac{7}{12}a_2(n)+\tfrac{9}{2}a_2\left(\tfrac{n}{3}\right)\\
&+192a_2\left(\tfrac{n}{6}\right)-\tfrac{16}{21}a_3(n)-\tfrac{64}{9}a_3\left(\tfrac{n}{2}\right)-\tfrac{14\,080}{63}a_3\left(\tfrac{n}{4}\right)\\
&+4a_4(n)+384a_5(n)-1152a_6(n)+512a_7(n),\\
&N(1^2,2^2,3^2,6^6;n)=-\tfrac{1}{91\,728}\sigma_5(n)+\tfrac{1}{91\,728}\sigma_5\left(\tfrac{n}{2}\right)\\
&-\tfrac{5}{5096}\sigma_5\left(\tfrac{n}{3}\right)-\tfrac{1}{5733}\sigma_5\left(\tfrac{n}{4}\right)+\tfrac{5}{5096}\sigma_5\left(\tfrac{n}{6}\right)+\tfrac{64}{5733}\sigma_5\left(\tfrac{n}{8}\right)\\
&-\tfrac{10}{637}\sigma_5\left(\tfrac{n}{12}\right)+\tfrac{640}{637}\sigma_5\left(\tfrac{n}{24}\right)-\tfrac{440}{819}a_1(n)+\tfrac{320}{273}a_1\left(\tfrac{n}{2}\right)\\
&+\tfrac{25\,264}{91}a_1\left(\tfrac{n}{4}\right)+\tfrac{374\,272}{273}a_1\left(\tfrac{n}{8}\right)+\tfrac{7}{6}a_2(n)+\tfrac{16}{3}a_2\left(\tfrac{n}{2}\right)\\
&-15a_2\left(\tfrac{n}{3}\right)+120a_2\left(\tfrac{n}{6}\right)-\tfrac{40}{63}a_3(n)-16a_3\left(\tfrac{n}{2}\right)\\
&-\tfrac{4096}{21}a_3\left(\tfrac{n}{4}\right)+4a_4(n)+36a_4\left(\tfrac{n}{3}\right)+448a_5(n)\\
&-1344a_6(n)+256a_7(n),\\
&N(1^2,2^2,3^4,6^4;n)=-\tfrac{1}{45\,864}\sigma_5(n)+\tfrac{1}{45\,864}\sigma_5\left(\tfrac{n}{2}\right)\\
&-\tfrac{5}{2548}\sigma_5\left(\tfrac{n}{3}\right)-\tfrac{1}{5733}\sigma_5\left(\tfrac{n}{4}\right)+\tfrac{5}{2548}\sigma_5\left(\tfrac{n}{6}\right)\\
&+\tfrac{64}{5733}\sigma_5\left(\tfrac{n}{8}\right)-\tfrac{10}{637}\sigma_5\left(\tfrac{n}{12}\right)+\tfrac{640}{637}\sigma_5\left(\tfrac{n}{24}\right)-\tfrac{118}{91}a_1(n)\\
&-\tfrac{2812}{819}a_1\left(\tfrac{n}{2}\right)+\tfrac{193\,888}{819}a_1\left(\tfrac{n}{4}\right)+\tfrac{234\,496}{273}a_1\left(\tfrac{n}{8}\right)+\tfrac{5}{3}a_2(n)\\
&+\tfrac{40}{3}a_2\left(\tfrac{n}{2}\right)-36a_2\left(\tfrac{n}{3}\right)+96a_2\left(\tfrac{n}{6}\right)-\tfrac{8}{21}a_3(n)\\
&-\tfrac{224}{9}a_3\left(\tfrac{n}{2}\right)-\tfrac{8704}{63}a_3\left(\tfrac{n}{4}\right)+4a_4(n)+72a_4\left(\tfrac{n}{3}\right)\\
&+512a_5\left(\tfrac{n}{2}\right)-1536a_6\left(\tfrac{n}{2}\right),\\
&N(1^2,2^2,3^6,6^2;n)=-\tfrac{1}{22\,932}\sigma_5(n)+\tfrac{1}{22\,932}\sigma_5\left(\tfrac{n}{2}\right)\\
&-\tfrac{5}{1274}\sigma_5\left(\tfrac{n}{3}\right)-\tfrac{1}{5733}\sigma_5\left(\tfrac{n}{4}\right)+\tfrac{5}{1274}\sigma_5\left(\tfrac{n}{6}\right)+\tfrac{64}{5733}\sigma_5\left(\tfrac{n}{8}\right)\\
&-\tfrac{10}{637}\sigma_5\left(\tfrac{n}{12}\right)+\tfrac{640}{637}\sigma_5\left(\tfrac{n}{24}\right)-\tfrac{1760}{819}a_1(n)-\tfrac{9992}{819}a_1\left(\tfrac{n}{2}\right)\\
&+\tfrac{68\,672}{819}a_1\left(\tfrac{n}{4}\right)-\tfrac{46\,080}{91}a_1\left(\tfrac{n}{8}\right)+2a_2(n)+24a_2\left(\tfrac{n}{2}\right)\\
&-60a_2\left(\tfrac{n}{3}\right)+96a_2\left(\tfrac{n}{6}\right)+\tfrac{8}{63}a_3(n)-\tfrac{304}{9}a_3\left(\tfrac{n}{2}\right)+\tfrac{256}{63}a_3\left(\tfrac{n}{4}\right)\\
&+4a_4(n)+108a_4\left(\tfrac{n}{3}\right)+512a_5(n)-1536a_6(n)-512a_7(n),\\
&N(1^2,2^2,3^8;n)=-\tfrac{1}{11\,466}\sigma_5(n)+\tfrac{1}{11\,466}\sigma_5\left(\tfrac{n}{2}\right)-\tfrac{5}{637}\sigma_5\left(\tfrac{n}{3}\right)\\
&-\tfrac{1}{5733}\sigma_5\left(\tfrac{n}{4}\right)+\tfrac{5}{637}\sigma_5\left(\tfrac{n}{6}\right)+\tfrac{64}{5733}\sigma_5\left(\tfrac{n}{8}\right)-\tfrac{10}{637}\sigma_5\left(\tfrac{n}{12}\right)\\
&+\tfrac{640}{637}\sigma_5\left(\tfrac{n}{24}\right)-\tfrac{688}{273}a_1(n)-\tfrac{19\,984}{819}a_1\left(\tfrac{n}{2}\right)-\tfrac{438\,016}{819}a_1\left(\tfrac{n}{4}\right)\\
&-\tfrac{512\,000}{91}a_1\left(\tfrac{n}{8}\right)+\tfrac{8}{3}a_2(n)+40a_2\left(\tfrac{n}{2}\right)-72a_2\left(\tfrac{n}{3}\right)-48a_2\left(\tfrac{n}{6}\right)\\
&+\tfrac{80}{21}a_3(n)-\tfrac{320}{9}a_3\left(\tfrac{n}{2}\right)+\tfrac{37\,888}{63}a_3\left(\tfrac{n}{4}\right)+144a_4\left(\tfrac{n}{3}\right)-2048a_7(n),\\
&N(1^2,2^4,6^6;n)=-\tfrac{1}{52\,416}\sigma_5(n)+\tfrac{1}{52\,416}\sigma_5\left(\tfrac{n}{2}\right)+\tfrac{3}{5824}\sigma_5\left(\tfrac{n}{3}\right)\\
&+\tfrac{1}{1638}\sigma_5\left(\tfrac{n}{4}\right)-\tfrac{3}{5824}\sigma_5\left(\tfrac{n}{6}\right)-\tfrac{32}{819}\sigma_5\left(\tfrac{n}{8}\right)-\tfrac{3}{182}\sigma_5\left(\tfrac{n}{12}\right)\\
&+\tfrac{96}{91}\sigma_5\left(\tfrac{n}{24}\right)+\tfrac{85}{117}a_1(n)+\tfrac{1982}{117}a_1\left(\tfrac{n}{2}\right)+\tfrac{51\,568}{117}a_1\left(\tfrac{n}{4}\right)\\
&+\tfrac{33\,024}{13}a_1\left(\tfrac{n}{8}\right)+\tfrac{25}{24}a_2(n)-8a_2\left(\tfrac{n}{2}\right)+\tfrac{159}{8}a_2\left(\tfrac{n}{3}\right)+72a_2\left(\tfrac{n}{6}\right)\\
&-\tfrac{16}{9}a_3(n)-\tfrac{32}{9}a_3\left(\tfrac{n}{2}\right)-\tfrac{2752}{9}a_3\left(\tfrac{n}{4}\right)\\
&+4a_4(n)+544a_5(n)-1248a_6(n)+640a_7(n),
\end{aligned}
$$

$$
N(1^2,2^4,3^2,6^4;n)=-\tfrac{1}{26\,208}\sigma_5(n)+\tfrac{1}{26\,208}\sigma_5\left(\tfrac{n}{2}\right)+\tfrac{3}{2912}\sigma_5\left(\tfrac{n}{3}\right)
$$

$$+\tfrac{1}{1638}\sigma_5\left(\tfrac{n}{4}\right)-\tfrac{3}{2912}\sigma_5\left(\tfrac{n}{6}\right)-\tfrac{32}{819}\sigma_5\left(\tfrac{n}{8}\right)-\tfrac{3}{182}\sigma_5\left(\tfrac{n}{12}\right)$$
$$+\tfrac{96}{91}\sigma_5\left(\tfrac{n}{24}\right)+\tfrac{404}{117}a_1(n)+\tfrac{3080}{117}a_1\left(\tfrac{n}{2}\right)+\tfrac{15\,872}{39}a_1\left(\tfrac{n}{4}\right)$$
$$+\tfrac{29\,696}{13}a_1\left(\tfrac{n}{8}\right)+\tfrac{1}{12}a_2(n)-28a_2\left(\tfrac{n}{2}\right)+\tfrac{375}{4}a_2\left(\tfrac{n}{3}\right)+372a_2\left(\tfrac{n}{6}\right)$$
$$-\tfrac{32}{9}a_3(n)+\tfrac{256}{9}a_3\left(\tfrac{n}{2}\right)-\tfrac{1024}{3}a_3\left(\tfrac{n}{4}\right)+4a_4(n)-108a_4\left(\tfrac{n}{3}\right)$$
$$+768a_5(n)-768a_6(n)+1024a_7(n),$$
$$N(1^2,2^4,3^4,6^2;n)=-\tfrac{1}{13\,104}\sigma_5(n)+\tfrac{1}{13\,104}\sigma_5\left(\tfrac{n}{2}\right)+\tfrac{3}{1456}\sigma_5\left(\tfrac{n}{3}\right)$$
$$+\tfrac{1}{1638}\sigma_5\left(\tfrac{n}{4}\right)-\tfrac{3}{1456}\sigma_5\left(\tfrac{n}{6}\right)-\tfrac{32}{819}\sigma_5\left(\tfrac{n}{8}\right)-\tfrac{3}{182}\sigma_5\left(\tfrac{n}{12}\right)$$
$$+\tfrac{96}{91}\sigma_5\left(\tfrac{n}{24}\right)+\tfrac{704}{117}a_1(n)+\tfrac{1724}{39}a_1\left(\tfrac{n}{2}\right)+\tfrac{55\,936}{117}a_1\left(\tfrac{n}{4}\right)$$
$$+\tfrac{36\,352}{13}a_1\left(\tfrac{n}{8}\right)-\tfrac{7}{6}a_2(n)-52a_2\left(\tfrac{n}{2}\right)+\tfrac{327}{2}a_2\left(\tfrac{n}{3}\right)$$
$$+492a_2\left(\tfrac{n}{6}\right)-\tfrac{44}{9}a_3(n)+\tfrac{160}{3}a_3\left(\tfrac{n}{2}\right)-\tfrac{3712}{9}a_3\left(\tfrac{n}{4}\right)$$
$$+4a_4(n)-216a_4\left(\tfrac{n}{3}\right)+832a_5(n)-192a_6(n)+1536a_7(n),$$
$$N(1^2,2^4,3^6;n)=-\tfrac{1}{6552}\sigma_5(n)+\tfrac{1}{6552}\sigma_5\left(\tfrac{n}{2}\right)+\tfrac{3}{728}\sigma_5\left(\tfrac{n}{3}\right)$$
$$+\tfrac{1}{1638}\sigma_5\left(\tfrac{n}{4}\right)-\tfrac{3}{728}\sigma_5\left(\tfrac{n}{6}\right)-\tfrac{32}{819}\sigma_5\left(\tfrac{n}{8}\right)-\tfrac{3}{182}\sigma_5\left(\tfrac{n}{12}\right)+\tfrac{96}{91}\sigma_5\left(\tfrac{n}{24}\right)$$
$$+\tfrac{322}{39}a_1(n)+\tfrac{8108}{117}a_1\left(\tfrac{n}{2}\right)+\tfrac{92\,960}{117}a_1\left(\tfrac{n}{4}\right)+\tfrac{69\,632}{13}a_1\left(\tfrac{n}{8}\right)-3a_2(n)$$
$$-76a_2\left(\tfrac{n}{2}\right)+225a_2\left(\tfrac{n}{3}\right)+420a_2\left(\tfrac{n}{6}\right)-\tfrac{16}{3}a_3(n)+\tfrac{592}{9}a_3\left(\tfrac{n}{2}\right)$$
$$-\tfrac{5888}{9}a_3\left(\tfrac{n}{4}\right)+4a_4(n)-324a_4\left(\tfrac{n}{3}\right)+512a_5(n)+2560a_7(n),$$

$$N(1^2,2^6,6^4;n)=-\tfrac{5}{91\,728}\sigma_5(n)+\tfrac{5}{91\,728}\sigma_5\left(\tfrac{n}{2}\right)-\tfrac{9}{20\,384}\sigma_5\left(\tfrac{n}{3}\right)$$
$$-\tfrac{10}{5733}\sigma_5\left(\tfrac{n}{4}\right)+\tfrac{9}{20\,384}\sigma_5\left(\tfrac{n}{6}\right)+\tfrac{640}{5733}\sigma_5\left(\tfrac{n}{8}\right)-\tfrac{9}{637}\sigma_5\left(\tfrac{n}{12}\right)$$
$$+\tfrac{576}{637}\sigma_5\left(\tfrac{n}{24}\right)+\tfrac{1076}{819}a_1(n)-\tfrac{1024}{819}a_1\left(\tfrac{n}{2}\right)+\tfrac{47\,392}{273}a_1\left(\tfrac{n}{4}\right)$$
$$+\tfrac{51\,712}{91}a_1\left(\tfrac{n}{8}\right)+\tfrac{11}{6}a_2(n)+\tfrac{141}{4}a_2\left(\tfrac{n}{3}\right)+480a_2\left(\tfrac{n}{6}\right)-\tfrac{200}{63}a_3(n)$$
$$+\tfrac{160}{9}a_3\left(\tfrac{n}{2}\right)-\tfrac{1408}{7}a_3\left(\tfrac{n}{4}\right)+4a_4(n)+960a_5(n)$$
$$-1344a_6(n)+256a_7(n),$$
$$N(1^2,2^6,3^2,6^2;n)=-\tfrac{5}{45\,864}\sigma_5(n)+\tfrac{5}{45\,864}\sigma_5\left(\tfrac{n}{2}\right)$$
$$-\tfrac{9}{10\,192}\sigma_5\left(\tfrac{n}{3}\right)-\tfrac{10}{5733}\sigma_5\left(\tfrac{n}{4}\right)+\tfrac{9}{10\,192}\sigma_5\left(\tfrac{n}{6}\right)+\tfrac{640}{5733}\sigma_5\left(\tfrac{n}{8}\right)$$
$$-\tfrac{9}{637}\sigma_5\left(\tfrac{n}{12}\right)+\tfrac{576}{637}\sigma_5\left(\tfrac{n}{24}\right)-\tfrac{5492}{819}a_1(n)-\tfrac{21\,704}{819}a_1\left(\tfrac{n}{2}\right)$$
$$+\tfrac{118\,736}{273}a_1\left(\tfrac{n}{4}\right)+\tfrac{238\,080}{91}a_1\left(\tfrac{n}{8}\right)+5a_2(n)+56a_2\left(\tfrac{n}{2}\right)$$
$$-\tfrac{363}{2}a_2\left(\tfrac{n}{3}\right)-336a_2\left(\tfrac{n}{6}\right)+\tfrac{104}{63}a_3(n)-\tfrac{688}{9}a_3\left(\tfrac{n}{2}\right)$$
$$-\tfrac{5120}{21}a_3\left(\tfrac{n}{4}\right)+4a_4(n)+324a_4\left(\tfrac{n}{3}\right)+576a_5(n)$$
$$-3264a_6(n)-256a_7(n),$$

$$N(1^2,2^6,3^4;n)=-\tfrac{5}{22\,932}\sigma_5(n)+\tfrac{5}{22\,932}\sigma_5\left(\tfrac{n}{2}\right)-\tfrac{9}{5096}\sigma_5\left(\tfrac{n}{3}\right)$$
$$-\tfrac{10}{5733}\sigma_5\left(\tfrac{n}{4}\right)+\tfrac{9}{5096}\sigma_5\left(\tfrac{n}{6}\right)+\tfrac{640}{5733}\sigma_5\left(\tfrac{n}{8}\right)-\tfrac{9}{637}\sigma_5\left(\tfrac{n}{12}\right)$$
$$+\tfrac{576}{637}\sigma_5\left(\tfrac{n}{24}\right)-\tfrac{12\,440}{819}a_1(n)-\tfrac{73\,256}{819}a_1\left(\tfrac{n}{2}\right)+\tfrac{126\,016}{273}a_1\left(\tfrac{n}{4}\right)$$
$$+\tfrac{377\,856}{91}a_1\left(\tfrac{n}{8}\right)+\tfrac{22}{3}a_2(n)+128a_2\left(\tfrac{n}{2}\right)-411a_2\left(\tfrac{n}{3}\right)$$
$$-840a_2\left(\tfrac{n}{6}\right)+\tfrac{488}{63}a_3(n)-\tfrac{1408}{9}a_3\left(\tfrac{n}{2}\right)-\tfrac{2304}{7}a_3\left(\tfrac{n}{4}\right)$$
$$+4a_4(n)+648a_4\left(\tfrac{n}{3}\right)-384a_5(n)-4992a_6(n)-1024a_7(n),$$
$$N(1^2,2^8,6^2;n)=-\tfrac{61}{366\,912}\sigma_5(n)+\tfrac{61}{366\,912}\sigma_5\left(\tfrac{n}{2}\right)+\tfrac{27}{40\,768}\sigma_5\left(\tfrac{n}{3}\right)$$
$$+\tfrac{61}{11\,466}\sigma_5\left(\tfrac{n}{4}\right)-\tfrac{27}{40\,768}\sigma_5\left(\tfrac{n}{6}\right)-\tfrac{1952}{5733}\sigma_5\left(\tfrac{n}{8}\right)-\tfrac{27}{1274}\sigma_5\left(\tfrac{n}{12}\right)$$
$$+\tfrac{864}{637}\sigma_5\left(\tfrac{n}{24}\right)+\tfrac{1207}{819}a_1(n)+\tfrac{34\,114}{819}a_1\left(\tfrac{n}{2}\right)+\tfrac{178\,832}{273}a_1\left(\tfrac{n}{4}\right)$$
$$+\tfrac{838\,400}{273}a_1\left(\tfrac{n}{8}\right)+\tfrac{17}{8}a_2(n)-\tfrac{40}{3}a_2\left(\tfrac{n}{2}\right)+\tfrac{321}{8}a_2\left(\tfrac{n}{3}\right)$$
$$+24a_2\left(\tfrac{n}{6}\right)-\tfrac{232}{63}a_3(n)-\tfrac{64}{9}a_3\left(\tfrac{n}{2}\right)-\tfrac{7232}{21}a_3\left(\tfrac{n}{4}\right)$$
$$+4a_4(n)+1248a_5(n)-1824a_6(n)+896a_7(n),$$
$$N(1^2,2^8,3^2;n)=-\tfrac{61}{183\,456}\sigma_5(n)+\tfrac{61}{183\,456}\sigma_5\left(\tfrac{n}{2}\right)+\tfrac{27}{20\,384}\sigma_5\left(\tfrac{n}{3}\right)$$

$$+\tfrac{61}{11\,466}\sigma_5\left(\tfrac{n}{4}\right)-\tfrac{27}{20\,384}\sigma_5\left(\tfrac{n}{6}\right)-\tfrac{1952}{5733}\sigma_5\left(\tfrac{n}{8}\right)-\tfrac{27}{1274}\sigma_5\left(\tfrac{n}{12}\right)$$
$$+\tfrac{864}{637}\sigma_5\left(\tfrac{n}{24}\right)+\tfrac{6568}{273}a_1(n)+\tfrac{27\,232}{273}a_1\left(\tfrac{n}{2}\right)-\tfrac{56\,384}{91}a_1\left(\tfrac{n}{4}\right)$$
$$-\tfrac{504\,832}{91}a_1\left(\tfrac{n}{8}\right)-\tfrac{39}{4}a_2(n)-180a_2\left(\tfrac{n}{2}\right)+\tfrac{2601}{4}a_2\left(\tfrac{n}{3}\right)$$
$$+2124a_2\left(\tfrac{n}{6}\right)-\tfrac{304}{21}a_3(n)+\tfrac{800}{3}a_3\left(\tfrac{n}{2}\right)+\tfrac{1024}{7}a_3\left(\tfrac{n}{4}\right)$$
$$+4a_4(n)-972a_4\left(\tfrac{n}{3}\right)+1152a_5(n)+5760a_6(n)+1536a_7(n),$$
$$N(1^2,2^{10};n)=-\tfrac{1}{2016}\sigma_5(n)+\tfrac{1}{2016}\sigma_5\left(\tfrac{n}{2}\right)-\tfrac{1}{63}\sigma_5\left(\tfrac{n}{4}\right)$$

$+\frac{64}{63}\sigma_5\left(\frac{n}{8}\right)-\frac{1}{4}a_2(n)+4a_4(n),$
$N(1^3,2,3,6^7;n)=-\frac{5}{366\,912}\sigma_5(n)+\frac{5}{366\,912}\sigma_5\left(\frac{n}{2}\right)+\frac{41}{40\,768}\sigma_5\left(\frac{n}{3}\right)$
$+\frac{5}{22\,932}\sigma_5\left(\frac{n}{4}\right)-\frac{41}{40\,768}\sigma_5\left(\frac{n}{6}\right)-\frac{80}{5733}\sigma_5\left(\frac{n}{8}\right)-\frac{41}{2548}\sigma_5\left(\frac{n}{12}\right)$
$+\frac{656}{637}\sigma_5\left(\frac{n}{24}\right)+\frac{1175}{1638}a_1(n)+\frac{10\,391}{819}a_1\left(\frac{n}{2}\right)+\frac{420\,848}{819}a_1\left(\frac{n}{4}\right)$
$+\frac{848\,384}{273}a_1\left(\frac{n}{8}\right)+\frac{23}{24}a_2(n)-\frac{22}{3}a_2\left(\frac{n}{2}\right)+\frac{159}{8}a_2\left(\frac{n}{3}\right)+234a_2\left(\frac{n}{6}\right)$
$-\frac{106}{63}a_3(n)-\frac{56}{9}a_3\left(\frac{n}{2}\right)-\frac{25\,856}{63}a_3\left(\frac{n}{4}\right)+6a_4(n)-18a_4\left(\frac{n}{3}\right)$
$+640a_5(n)-1728a_6(n)+896a_7(n),$
$N(1^3,2,3^3,6^5;n)=-\frac{5}{183\,456}\sigma_5(n)+\frac{5}{183\,456}\sigma_5\left(\frac{n}{2}\right)+\frac{41}{20\,384}\sigma_5\left(\frac{n}{3}\right)$
$+\frac{5}{22\,932}\sigma_5\left(\frac{n}{4}\right)-\frac{41}{20\,384}\sigma_5\left(\frac{n}{6}\right)-\frac{80}{5733}\sigma_5\left(\frac{n}{8}\right)-\frac{41}{2548}\sigma_5\left(\frac{n}{12}\right)$
$+\frac{656}{637}\sigma_5\left(\frac{n}{24}\right)+\frac{1448}{819}a_1(n)+\frac{1662}{91}a_1\left(\frac{n}{2}\right)+\frac{436\,136}{819}a_1\left(\frac{n}{4}\right)$
$+\frac{848\,384}{273}a_1\left(\frac{n}{8}\right)+\frac{11}{12}a_2(n)-\frac{46}{3}a_2\left(\frac{n}{2}\right)+\frac{195}{4}a_2\left(\frac{n}{3}\right)$
$+330a_2\left(\frac{n}{6}\right)-\frac{170}{63}a_3(n)+\frac{16}{3}a_3\left(\frac{n}{2}\right)-\frac{27\,200}{63}a_3\left(\frac{n}{4}\right)+6a_4(n)$
$-54a_4\left(\frac{n}{3}\right)+800a_5(n)-1632a_6(n)+1024a_7(n),$

$N(1^3,2,3^5,6^3;n)=-\frac{5}{91\,728}\sigma_5(n)+\frac{5}{91\,728}\sigma_5\left(\frac{n}{2}\right)+\frac{41}{10\,192}\sigma_5\left(\frac{n}{3}\right)$
$+\frac{5}{22\,932}\sigma_5\left(\frac{n}{4}\right)-\frac{41}{10\,192}\sigma_5\left(\frac{n}{6}\right)-\frac{80}{5733}\sigma_5\left(\frac{n}{8}\right)-\frac{41}{2548}\sigma_5\left(\frac{n}{12}\right)$
$+\frac{656}{637}\sigma_5\left(\frac{n}{24}\right)+\frac{4427}{1638}a_1(n)+\frac{21\,271}{819}a_1\left(\frac{n}{2}\right)+\frac{163\,336}{273}a_1\left(\frac{n}{4}\right)$
$+\frac{964\,864}{273}a_1\left(\frac{n}{8}\right)+\frac{2}{3}a_2(n)-\frac{74}{3}a_2\left(\frac{n}{2}\right)+75a_2\left(\frac{n}{3}\right)+366a_2\left(\frac{n}{6}\right)$
$-\frac{214}{63}a_3(n)+\frac{128}{9}a_3\left(\frac{n}{2}\right)-\frac{10\,112}{21}a_3\left(\frac{n}{4}\right)+6a_4(n)-90a_4\left(\frac{n}{3}\right)$
$+896a_5(n)-1536a_6(n)+1280a_7(n),$
$N(1^3,2,3^7,6;n)=-\frac{5}{45\,864}\sigma_5(n)+\frac{5}{45\,864}\sigma_5\left(\frac{n}{2}\right)+\frac{41}{5096}\sigma_5\left(\frac{n}{3}\right)$
$+\frac{5}{22\,932}\sigma_5\left(\frac{n}{4}\right)-\frac{41}{5096}\sigma_5\left(\frac{n}{6}\right)-\frac{80}{5733}\sigma_5\left(\frac{n}{8}\right)-\frac{41}{2548}\sigma_5\left(\frac{n}{12}\right)$
$+\frac{656}{637}\sigma_5\left(\frac{n}{24}\right)+\frac{2789}{819}a_1(n)+\frac{9388}{273}a_1\left(\frac{n}{2}\right)+\frac{630\,512}{819}a_1\left(\frac{n}{4}\right)$
$+\frac{453\,632}{91}a_1\left(\frac{n}{8}\right)-34a_2\left(\frac{n}{2}\right)+96a_2\left(\frac{n}{3}\right)+330a_2\left(\frac{n}{6}\right)$
$-\frac{218}{63}a_3(n)+\frac{56}{3}a_3\left(\frac{n}{2}\right)-\frac{39\,296}{63}a_3\left(\frac{n}{4}\right)+6a_4(n)-126a_4\left(\frac{n}{3}\right)$
$+768a_5(n)-1536a_6(n)+1792a_7(n),$
$N(1^3,2^3,3,6^5;n)=-\frac{1}{28\,224}\sigma_5(n)+\frac{1}{28\,224}\sigma_5\left(\frac{n}{2}\right)-\frac{3}{3136}\sigma_5\left(\frac{n}{3}\right)$
$-\frac{1}{1764}\sigma_5\left(\frac{n}{4}\right)+\frac{3}{3136}\sigma_5\left(\frac{n}{6}\right)+\frac{16}{441}\sigma_5\left(\frac{n}{8}\right)-\frac{3}{196}\sigma_5\left(\frac{n}{12}\right)$
$+\frac{48}{49}\sigma_5\left(\frac{n}{24}\right)-\frac{31}{126}a_1(n)+\frac{29}{7}a_1\left(\frac{n}{2}\right)+\frac{26\,912}{63}a_1\left(\frac{n}{4}\right)+\frac{42\,496}{21}a_1\left(\frac{n}{8}\right)$
$+\frac{55}{24}a_2(n)+\frac{22}{3}a_2\left(\frac{n}{2}\right)-\frac{57}{8}a_2\left(\frac{n}{3}\right)+246a_2\left(\frac{n}{6}\right)-\frac{130}{63}a_3(n)$
$-\frac{40}{3}a_3\left(\frac{n}{2}\right)-\frac{19\,328}{63}a_3\left(\frac{n}{4}\right)+6a_4(n)+54a_4\left(\frac{n}{3}\right)+896a_5(n)$
$-2112a_6(n)+384a_7(n),$
$N(1^3,2^3,3^3,6^3;n)=-\frac{1}{14\,112}\sigma_5(n)+\frac{1}{14\,112}\sigma_5\left(\frac{n}{2}\right)-\frac{3}{1568}\sigma_5\left(\frac{n}{3}\right)$
$-\frac{1}{1764}\sigma_5\left(\frac{n}{4}\right)+\frac{3}{1568}\sigma_5\left(\frac{n}{6}\right)+\frac{16}{441}\sigma_5\left(\frac{n}{8}\right)-\frac{3}{196}\sigma_5\left(\frac{n}{12}\right)+\frac{48}{49}\sigma_5\left(\frac{n}{24}\right)$
$-\frac{178}{63}a_1(n)-\frac{54}{7}a_1\left(\frac{n}{2}\right)+\frac{26\,968}{63}a_1\left(\frac{n}{4}\right)+\frac{38\,912}{21}a_1\left(\frac{n}{8}\right)+\frac{43}{12}a_2(n)$
$+\frac{86}{3}a_2\left(\frac{n}{2}\right)-\frac{309}{4}a_2\left(\frac{n}{3}\right)+78a_2\left(\frac{n}{6}\right)-\frac{50}{63}a_3(n)-\frac{128}{3}a_3\left(\frac{n}{2}\right)$
$-\frac{16\,192}{63}a_3\left(\frac{n}{4}\right)+6a_4(n)+162a_4\left(\frac{n}{3}\right)+928a_5(n)-2784a_6(n),$
$N(1^3,2^3,3^5,6;n)=-\frac{1}{7056}\sigma_5(n)+\frac{1}{7056}\sigma_5\left(\frac{n}{2}\right)-\frac{3}{784}\sigma_5\left(\frac{n}{3}\right)$
$-\frac{1}{1764}\sigma_5\left(\frac{n}{4}\right)+\frac{3}{784}\sigma_5\left(\frac{n}{6}\right)+\frac{16}{441}\sigma_5\left(\frac{n}{8}\right)-\frac{3}{196}\sigma_5\left(\frac{n}{12}\right)+\frac{48}{49}\sigma_5\left(\frac{n}{24}\right)$
$-\frac{719}{126}a_1(n)-\frac{709}{21}a_1\left(\frac{n}{2}\right)+\frac{14\,536}{63}a_1\left(\frac{n}{4}\right)+\frac{12\,032}{21}a_1\left(\frac{n}{8}\right)+\frac{13}{3}a_2(n)$
$+\frac{170}{3}a_2\left(\frac{n}{2}\right)-156a_2\left(\frac{n}{3}\right)-30a_2\left(\frac{n}{6}\right)+\frac{82}{63}a_3(n)-\frac{208}{3}a_3\left(\frac{n}{2}\right)$
$-\frac{6784}{63}a_3\left(\frac{n}{4}\right)+6a_4(n)+270a_4\left(\frac{n}{3}\right)+640a_5(n)-3072a_6(n)$
$-768a_7(n),$

$N(1^3,2^5,3,6^3;n)=-\frac{41}{366\,912}\sigma_5(n)+\frac{41}{366\,912}\sigma_5\left(\frac{n}{2}\right)+\frac{45}{40\,768}\sigma_5\left(\frac{n}{3}\right)$
$+\frac{41}{22\,932}\sigma_5\left(\frac{n}{4}\right)-\frac{45}{40\,768}\sigma_5\left(\frac{n}{6}\right)-\frac{656}{5733}\sigma_5\left(\frac{n}{8}\right)-\frac{45}{2548}\sigma_5\left(\frac{n}{12}\right)$
$+\frac{720}{637}\sigma_5\left(\frac{n}{24}\right)+\frac{9271}{1638}a_1(n)+\frac{36\,503}{819}a_1\left(\frac{n}{2}\right)+\frac{166\,256}{273}a_1\left(\frac{n}{4}\right)$
$+\frac{262\,656}{91}a_1\left(\frac{n}{8}\right)+\frac{19}{24}a_2(n)-42a_2\left(\frac{n}{2}\right)+\frac{1227}{8}a_2\left(\frac{n}{3}\right)+642a_2\left(\frac{n}{6}\right)$
$-\frac{410}{63}a_3(n)+\frac{424}{9}a_3\left(\frac{n}{2}\right)-\frac{3328}{7}a_3\left(\frac{n}{4}\right)+6a_4(n)-162a_4\left(\frac{n}{3}\right)$
$+1536a_5(n)-1344a_6(n)+1408a_7(n),$
$N(1^3,2^5,3^3,6;n)=-\frac{41}{183\,456}\sigma_5(n)+\frac{41}{183\,456}\sigma_5\left(\frac{n}{2}\right)+\frac{45}{20\,384}\sigma_5\left(\frac{n}{3}\right)$

$$+\frac{41}{22\,932}\sigma_5\left(\frac{n}{4}\right)-\frac{45}{20\,384}\sigma_5\left(\frac{n}{6}\right)-\frac{656}{5733}\sigma_5\left(\frac{n}{8}\right)-\frac{45}{2548}\sigma_5\left(\frac{n}{12}\right)$$
$$+\frac{720}{637}\sigma_5\left(\frac{n}{24}\right)+\frac{10\,636}{819}a_1(n)+\frac{64\,270}{819}a_1\left(\frac{n}{2}\right)+\frac{110\,200}{273}a_1\left(\frac{n}{4}\right)$$
$$+\frac{415\,232}{273}a_1\left(\frac{n}{8}\right)-\frac{41}{12}a_2(n)-\frac{310}{3}a_2\left(\frac{n}{2}\right)+\frac{1407}{4}a_2\left(\frac{n}{3}\right)+1122a_2\left(\frac{n}{6}\right)$$
$$-\frac{610}{63}a_3(n)+\frac{1136}{9}a_3\left(\frac{n}{2}\right)-\frac{2880}{7}a_3\left(\frac{n}{4}\right)+6a_4(n)-486a_4\left(\frac{n}{3}\right)$$
$$+1440a_5(n)+1056a_6(n)+2048a_7(n),$$
$$N(1^3,2^7,3,6;n)=-\frac{121}{366\,912}\sigma_5(n)+\frac{121}{366\,912}\sigma_5\left(\frac{n}{2}\right)-\frac{27}{40\,768}\sigma_5\left(\frac{n}{3}\right)$$
$$-\frac{121}{22\,932}\sigma_5\left(\frac{n}{4}\right)+\frac{27}{40\,768}\sigma_5\left(\frac{n}{6}\right)+\frac{1936}{5733}\sigma_5\left(\frac{n}{8}\right)-\frac{27}{2548}\sigma_5\left(\frac{n}{12}\right)$$
$$+\frac{432}{637}\sigma_5\left(\frac{n}{24}\right)-\frac{16\,883}{1638}a_1(n)-\frac{24\,559}{819}a_1\left(\frac{n}{2}\right)+\frac{209\,536}{273}a_1\left(\frac{n}{4}\right)$$
$$+\frac{1292\,800}{273}a_1\left(\frac{n}{8}\right)+\frac{57}{8}a_2(n)+\frac{238}{3}a_2\left(\frac{n}{2}\right)-\frac{2229}{8}a_2\left(\frac{n}{3}\right)$$
$$-690a_2\left(\frac{n}{6}\right)+\frac{190}{63}a_3(n)-\frac{1112}{9}a_3\left(\frac{n}{2}\right)-\frac{8320}{21}a_3\left(\frac{n}{4}\right)+6a_4(n)$$
$$+486a_4\left(\frac{n}{3}\right)+768a_5(n)-4800a_6(n)-128a_7(n),$$
$$N(1^4,6^8;n)=-\frac{1}{91\,728}\sigma_5(n)+\frac{1}{91\,728}\sigma_5\left(\frac{n}{2}\right)-\frac{5}{5096}\sigma_5\left(\frac{n}{3}\right)$$
$$-\frac{1}{5733}\sigma_5\left(\frac{n}{4}\right)+\frac{5}{5096}\sigma_5\left(\frac{n}{6}\right)+\frac{64}{5733}\sigma_5\left(\frac{n}{8}\right)-\frac{10}{637}\sigma_5\left(\frac{n}{12}\right)$$
$$+\frac{640}{637}\sigma_5\left(\frac{n}{24}\right)+\frac{32}{91}a_1(n)+\frac{6784}{819}a_1\left(\frac{n}{2}\right)+\frac{164\,608}{273}a_1\left(\frac{n}{4}\right)$$
$$+\frac{1073\,152}{273}a_1\left(\frac{n}{8}\right)+\frac{7}{6}a_2(n)-\frac{8}{3}a_2\left(\frac{n}{2}\right)+9a_2\left(\frac{n}{3}\right)+336a_2\left(\frac{n}{6}\right)$$
$$-\frac{32}{21}a_3(n)-\frac{64}{9}a_3\left(\frac{n}{2}\right)-\frac{11\,264}{21}a_3\left(\frac{n}{4}\right)+8a_4(n)+768a_5(n)$$
$$-2304a_6(n)+1024a_7(n),$$

$$N(1^4,3^2,6^6;n)=-\frac{1}{45\,864}\sigma_5(n)+\frac{1}{45\,864}\sigma_5\left(\frac{n}{2}\right)-\frac{5}{2548}\sigma_5\left(\frac{n}{3}\right)$$
$$-\frac{1}{5733}\sigma_5\left(\frac{n}{4}\right)+\frac{5}{2548}\sigma_5\left(\frac{n}{6}\right)+\frac{64}{5733}\sigma_5\left(\frac{n}{8}\right)-\frac{10}{637}\sigma_5\left(\frac{n}{12}\right)$$
$$+\frac{640}{637}\sigma_5\left(\frac{n}{24}\right)-\frac{152}{819}a_1(n)+\frac{7016}{819}a_1\left(\frac{n}{2}\right)+\frac{552\,064}{819}a_1\left(\frac{n}{4}\right)$$
$$+\frac{1166\,336}{273}a_1\left(\frac{n}{8}\right)+\frac{7}{3}a_2(n)+\frac{8}{3}a_2\left(\frac{n}{2}\right)-6a_2\left(\frac{n}{3}\right)+288a_2\left(\frac{n}{6}\right)$$
$$-\frac{136}{63}a_3(n)-\frac{128}{9}a_3\left(\frac{n}{2}\right)-\frac{35\,584}{63}a_3\left(\frac{n}{4}\right)+8a_4(n)+36a_4\left(\frac{n}{3}\right)$$
$$+896a_5(n)-2688a_6(n)+1024a_7(n),$$
$$N(1^4,3^4,6^4;n)=-\frac{1}{22\,932}\sigma_5(n)+\frac{1}{22\,932}\sigma_5\left(\frac{n}{2}\right)-\frac{5}{1274}\sigma_5\left(\frac{n}{3}\right)$$
$$-\frac{1}{5733}\sigma_5\left(\frac{n}{4}\right)+\frac{5}{1274}\sigma_5\left(\frac{n}{6}\right)+\frac{64}{5733}\sigma_5\left(\frac{n}{8}\right)-\frac{10}{637}\sigma_5\left(\frac{n}{12}\right)$$
$$+\frac{640}{637}\sigma_5\left(\frac{n}{24}\right)-\frac{668}{819}a_1(n)+\frac{3112}{819}a_1\left(\frac{n}{2}\right)+\frac{575\,360}{819}a_1\left(\frac{n}{4}\right)$$
$$+\frac{419\,840}{91}a_1\left(\frac{n}{8}\right)+\frac{10}{3}a_2(n)+8a_2\left(\frac{n}{2}\right)-24a_2\left(\frac{n}{3}\right)+240a_2\left(\frac{n}{6}\right)$$
$$-\frac{160}{63}a_3(n)-\frac{160}{9}a_3\left(\frac{n}{2}\right)-\frac{37\,376}{63}a_3\left(\frac{n}{4}\right)+8a_4(n)+72a_4\left(\frac{n}{3}\right)$$
$$+1024a_5(n)-3072a_6(n)+1024a_7(n),$$
$$N(1^4,3^6,6^2;n)=-\frac{1}{11\,466}\sigma_5(n)+\frac{1}{11\,466}\sigma_5\left(\frac{n}{2}\right)-\frac{5}{637}\sigma_5\left(\frac{n}{3}\right)$$
$$-\frac{1}{5733}\sigma_5\left(\frac{n}{4}\right)+\frac{5}{637}\sigma_5\left(\frac{n}{6}\right)+\frac{64}{5733}\sigma_5\left(\frac{n}{8}\right)-\frac{10}{637}\sigma_5\left(\frac{n}{12}\right)$$
$$+\frac{640}{637}\sigma_5\left(\frac{n}{24}\right)-\frac{1336}{819}a_1(n)-\frac{2536}{273}a_1\left(\frac{n}{2}\right)+\frac{505\,472}{819}a_1\left(\frac{n}{4}\right)$$
$$+\frac{419\,840}{91}a_1\left(\frac{n}{8}\right)+4a_2(n)+8a_2\left(\frac{n}{2}\right)-12a_2\left(\frac{n}{3}\right)+240a_2\left(\frac{n}{6}\right)$$
$$-\frac{152}{63}a_3(n)-\frac{32}{3}a_3\left(\frac{n}{2}\right)-\frac{37\,376}{63}a_3\left(\frac{n}{4}\right)+8a_4(n)+72a_4\left(\frac{n}{3}\right)$$
$$+1024a_5(n)-3072a_6(n)+1024a_7(n),$$

$$N(1^4,3^8;n)=-\frac{1}{5733}\sigma_5(n)-\frac{10}{637}\sigma_5\left(\frac{n}{3}\right)+\frac{64}{5733}\sigma_5\left(\frac{n}{4}\right)+\frac{640}{637}\sigma_5\left(\frac{n}{12}\right)$$
$$-\frac{1216}{819}a_1(n)-\frac{2048}{63}a_1\left(\frac{n}{2}\right)-\frac{45\,056}{273}a_1\left(\frac{n}{4}\right)+\frac{16}{3}a_2(n)+96a_2\left(\frac{n}{3}\right)$$
$$+\frac{256}{63}a_3(n)+\frac{2048}{63}a_3\left(\frac{n}{2}\right),$$

$$N(1^4,2^2,6^6;n)=-\frac{1}{26\,208}\sigma_5(n)+\frac{1}{26\,208}\sigma_5\left(\frac{n}{2}\right)+\frac{3}{2912}\sigma_5\left(\frac{n}{3}\right)$$
$$+\frac{1}{1638}\sigma_5\left(\frac{n}{4}\right)-\frac{3}{2912}\sigma_5\left(\frac{n}{6}\right)-\frac{32}{819}\sigma_5\left(\frac{n}{8}\right)-\frac{3}{182}\sigma_5\left(\frac{n}{12}\right)+\frac{96}{91}\sigma_5\left(\frac{n}{24}\right)$$
$$+\frac{170}{117}a_1(n)+\frac{3028}{117}a_1\left(\frac{n}{2}\right)+\frac{91\,088}{117}a_1\left(\frac{n}{4}\right)+\frac{175\,616}{39}a_1\left(\frac{n}{8}\right)+\frac{25}{12}a_2(n)$$
$$-\frac{28}{3}a_2\left(\frac{n}{2}\right)+\frac{159}{4}a_2\left(\frac{n}{3}\right)+252a_2\left(\frac{n}{6}\right)-\frac{32}{9}a_3(n)-\frac{16}{9}a_3\left(\frac{n}{2}\right)$$
$$-\frac{5120}{9}a_3\left(\frac{n}{4}\right)+8a_4(n)+1088a_5(n)-2496a_6(n)+1280a_7(n),$$
$$N(1^4,2^2,3^2,6^4;n)=-\frac{1}{13\,104}\sigma_5(n)+\frac{1}{13\,104}\sigma_5\left(\frac{n}{2}\right)+\frac{3}{1456}\sigma_5\left(\frac{n}{3}\right)$$

$$+\frac{1}{1638}\sigma_5\left(\frac{n}{4}\right)-\frac{3}{1456}\sigma_5\left(\frac{n}{6}\right)-\frac{32}{819}\sigma_5\left(\frac{n}{8}\right)-\frac{3}{182}\sigma_5\left(\frac{n}{12}\right)+\frac{96}{91}\sigma_5\left(\frac{n}{24}\right)$$
$$+\frac{496}{117}a_1(n)+\frac{1412}{39}a_1\left(\frac{n}{2}\right)+\frac{87\,968}{117}a_1\left(\frac{n}{4}\right)+\frac{162\,304}{39}a_1\left(\frac{n}{8}\right)+\frac{3}{2}a_2(n)$$

$-\frac{92}{3}a_2\left(\frac{n}{2}\right)+\frac{231}{2}a_2\left(\frac{n}{3}\right)+588a_2\left(\frac{n}{6}\right)-\frac{52}{9}a_3(n)+32a_3\left(\frac{n}{2}\right)$
$-\frac{5504}{9}a_3\left(\frac{n}{4}\right)+8a_4(n)-108a_4\left(\frac{n}{3}\right)+1472a_5(n)-2112a_6(n)$
$+1536a_7(n),$

$N(1^4,2^2,3^4,6^2;n)=-\frac{1}{6552}\sigma_5(n)+\frac{1}{6552}\sigma_5\left(\frac{n}{2}\right)+\frac{3}{728}\sigma_5\left(\frac{n}{3}\right)$
$+\frac{1}{1638}\sigma_5\left(\frac{n}{4}\right)-\frac{3}{728}\sigma_5\left(\frac{n}{6}\right)-\frac{32}{819}\sigma_5\left(\frac{n}{8}\right)-\frac{3}{182}\sigma_5\left(\frac{n}{12}\right)+\frac{96}{91}\sigma_5\left(\frac{n}{24}\right)$
$+\frac{784}{117}a_1(n)+\frac{6184}{117}a_1\left(\frac{n}{2}\right)+\frac{10\,560}{13}a_1\left(\frac{n}{4}\right)+\frac{182\,272}{39}a_1\left(\frac{n}{8}\right)+\frac{1}{3}a_2(n)$
$-\frac{164}{3}a_2\left(\frac{n}{2}\right)+183a_2\left(\frac{n}{3}\right)+708a_2\left(\frac{n}{6}\right)-\frac{64}{9}a_3(n)+\frac{512}{9}a_3\left(\frac{n}{2}\right)$
$-\frac{2048}{3}a_3\left(\frac{n}{4}\right)+8a_4(n)-216a_4\left(\frac{n}{3}\right)+1536a_5(n)-1536a_6(n)+2048a_7(n),$

$N(1^4,2^2,3^6;n)=-\frac{1}{3276}\sigma_5(n)+\frac{1}{3276}\sigma_5\left(\frac{n}{2}\right)+\frac{3}{364}\sigma_5\left(\frac{n}{3}\right)+\frac{1}{1638}\sigma_5\left(\frac{n}{4}\right)$
$-\frac{3}{364}\sigma_5\left(\frac{n}{6}\right)-\frac{32}{819}\sigma_5\left(\frac{n}{8}\right)-\frac{3}{182}\sigma_5\left(\frac{n}{12}\right)+\frac{96}{91}\sigma_5\left(\frac{n}{24}\right)+\frac{358}{39}a_1(n)$
$+\frac{8624}{117}a_1\left(\frac{n}{2}\right)+\frac{97\,952}{117}a_1\left(\frac{n}{4}\right)+\frac{69\,632}{13}a_1\left(\frac{n}{8}\right)-\frac{4}{3}a_2(n)-76a_2\left(\frac{n}{2}\right)$
$+252a_2\left(\frac{n}{3}\right)+420a_2\left(\frac{n}{6}\right)-4a_3(n)+\frac{688}{9}a_3\left(\frac{n}{2}\right)-\frac{5888}{9}a_3\left(\frac{n}{4}\right)$
$+4a_4(n)-324a_4\left(\frac{n}{3}\right)+512a_5(n)+2560a_7(n),$
$N(1^4,2^4,6^4;n)=-\frac{5}{45\,864}\sigma_5(n)+\frac{5}{45\,864}\sigma_5\left(\frac{n}{2}\right)-\frac{9}{10\,192}\sigma_5\left(\frac{n}{3}\right)$

$-\frac{10}{5733}\sigma_5\left(\frac{n}{4}\right)+\frac{9}{10\,192}\sigma_5\left(\frac{n}{6}\right)+\frac{640}{5733}\sigma_5\left(\frac{n}{8}\right)-\frac{9}{637}\sigma_5\left(\frac{n}{12}\right)+\frac{576}{637}\sigma_5\left(\frac{n}{24}\right)$
$+\frac{2152}{819}a_1(n)+\frac{11\,056}{819}a_1\left(\frac{n}{2}\right)+\frac{45\,888}{91}a_1\left(\frac{n}{4}\right)+\frac{481\,280}{273}a_1\left(\frac{n}{8}\right)+\frac{11}{3}a_2(n)$
$-\frac{16}{3}a_2\left(\frac{n}{2}\right)+\frac{141}{2}a_2\left(\frac{n}{3}\right)+744a_2\left(\frac{n}{6}\right)-\frac{400}{63}a_3(n)+\frac{224}{9}a_3\left(\frac{n}{2}\right)$
$-\frac{8704}{21}a_3\left(\frac{n}{4}\right)+8a_4(n)+1920a_5(n)-2688a_6(n)+512a_7(n),$
$N(1^4,2^4,3^2,6^2;n)=-\frac{5}{22\,932}\sigma_5(n)+\frac{5}{22\,932}\sigma_5\left(\frac{n}{2}\right)-\frac{9}{5096}\sigma_5\left(\frac{n}{3}\right)$
$-\frac{10}{5733}\sigma_5\left(\frac{n}{4}\right)+\frac{9}{5096}\sigma_5\left(\frac{n}{6}\right)+\frac{640}{5733}\sigma_5\left(\frac{n}{8}\right)-\frac{9}{637}\sigma_5\left(\frac{n}{12}\right)+\frac{576}{637}\sigma_5\left(\frac{n}{24}\right)$
$-\frac{1538}{273}a_1(n)-\frac{3428}{273}a_1\left(\frac{n}{2}\right)+\frac{66\,272}{91}a_1\left(\frac{n}{4}\right)+\frac{947\,200}{273}a_1\left(\frac{n}{8}\right)+\frac{20}{3}a_2(n)$
$+\frac{160}{3}a_2\left(\frac{n}{2}\right)-153a_2\left(\frac{n}{3}\right)-72a_2\left(\frac{n}{6}\right)-\frac{8}{7}a_3(n)-\frac{224}{3}a_3\left(\frac{n}{2}\right)$
$-\frac{8704}{21}a_3\left(\frac{n}{4}\right)+8a_4(n)+324a_4\left(\frac{n}{3}\right)+1536a_5(n)-4608a_6(n),$
$N(1^4,2^4,3^4;n)=-\frac{5}{11\,466}\sigma_5(n)+\frac{5}{11\,466}\sigma_5\left(\frac{n}{2}\right)-\frac{9}{2548}\sigma_5\left(\frac{n}{3}\right)$
$-\frac{10}{5733}\sigma_5\left(\frac{n}{4}\right)+\frac{9}{2548}\sigma_5\left(\frac{n}{6}\right)+\frac{640}{5733}\sigma_5\left(\frac{n}{8}\right)-\frac{9}{637}\sigma_5\left(\frac{n}{12}\right)$
$+\frac{576}{637}\sigma_5\left(\frac{n}{24}\right)-\frac{12\,140}{819}a_1(n)-\frac{67\,160}{819}a_1\left(\frac{n}{2}\right)+\frac{178\,432}{273}a_1\left(\frac{n}{4}\right)$
$+\frac{1226\,752}{273}a_1\left(\frac{n}{8}\right)+8a_2(n)+\frac{400}{3}a_2\left(\frac{n}{2}\right)-402a_2\left(\frac{n}{3}\right)$
$-696a_2\left(\frac{n}{6}\right)+\frac{416}{63}a_3(n)-\frac{1504}{9}a_3\left(\frac{n}{2}\right)-\frac{8704}{21}a_3\left(\frac{n}{4}\right)$
$+8a_4(n)+648a_4\left(\frac{n}{3}\right)-6144a_6(n)-1024a_7(n),$

$N(1^4,2^6,6^2;n)=-\frac{61}{183\,456}\sigma_5(n)+\frac{61}{183\,456}\sigma_5\left(\frac{n}{2}\right)+\frac{27}{20\,384}\sigma_5\left(\frac{n}{3}\right)$
$+\frac{61}{11\,466}\sigma_5\left(\frac{n}{4}\right)-\frac{27}{20\,384}\sigma_5\left(\frac{n}{6}\right)-\frac{1952}{5733}\sigma_5\left(\frac{n}{8}\right)-\frac{27}{1274}\sigma_5\left(\frac{n}{12}\right)$
$+\frac{864}{637}\sigma_5\left(\frac{n}{24}\right)+\frac{2414}{819}a_1(n)+\frac{48\,572}{819}a_1\left(\frac{n}{2}\right)+\frac{312\,784}{273}a_1\left(\frac{n}{4}\right)$
$+\frac{1513\,984}{273}a_1\left(\frac{n}{8}\right)+\frac{17}{4}a_2(n)-\frac{68}{3}a_2\left(\frac{n}{2}\right)+\frac{321}{4}a_2\left(\frac{n}{3}\right)+372a_2\left(\frac{n}{6}\right)$
$-\frac{464}{63}a_3(n)+\frac{16}{9}a_3\left(\frac{n}{2}\right)-\frac{14\,848}{21}a_3\left(\frac{n}{4}\right)+8a_4(n)+2496a_5(n)$
$-3648a_6(n)+1792a_7(n),$

$N(1^4,2^6,3^2;n)=-\frac{61}{91\,728}\sigma_5(n)+\frac{61}{91\,728}\sigma_5\left(\frac{n}{2}\right)+\frac{27}{10\,192}\sigma_5\left(\frac{n}{3}\right)$
$+\frac{61}{11\,466}\sigma_5\left(\frac{n}{4}\right)-\frac{27}{10\,192}\sigma_5\left(\frac{n}{6}\right)-\frac{1952}{5733}\sigma_5\left(\frac{n}{8}\right)-\frac{27}{1274}\sigma_5\left(\frac{n}{12}\right)$
$+\frac{864}{637}\sigma_5\left(\frac{n}{24}\right)+\frac{19\,934}{819}a_1(n)+\frac{85\,496}{819}a_1\left(\frac{n}{2}\right)-\frac{23\,552}{273}a_1\left(\frac{n}{4}\right)$
$-\frac{442\,880}{273}a_1\left(\frac{n}{8}\right)-\frac{19}{2}a_2(n)-\frac{548}{3}a_2\left(\frac{n}{2}\right)+\frac{1317}{2}a_2\left(\frac{n}{3}\right)$
$+2244a_2\left(\frac{n}{6}\right)-\frac{956}{63}a_3(n)+\frac{2368}{9}a_3\left(\frac{n}{2}\right)-\frac{6784}{21}a_3\left(\frac{n}{4}\right)$
$+8a_4(n)-972a_4\left(\frac{n}{3}\right)+1344a_5(n)+4416a_6(n)+2560a_7(n),$
$N(1^4,2^8;n)=-\frac{1}{1008}\sigma_5(n)+\frac{1}{1008}\sigma_5\left(\frac{n}{2}\right)-\frac{1}{63}\sigma_5\left(\frac{n}{4}\right)+\frac{64}{63}\sigma_5\left(\frac{n}{8}\right)$
$-\frac{1}{2}a_2(n)-8a_2\left(\frac{n}{2}\right)+8a_4(n),$
$N(1^5,2,3,6^5;n)=-\frac{1}{14\,112}\sigma_5(n)+\frac{1}{14\,112}\sigma_5\left(\frac{n}{2}\right)-\frac{3}{1568}\sigma_5\left(\frac{n}{3}\right)$
$-\frac{1}{1764}\sigma_5\left(\frac{n}{4}\right)+\frac{3}{1568}\sigma_5\left(\frac{n}{6}\right)+\frac{16}{441}\sigma_5\left(\frac{n}{8}\right)-\frac{3}{196}\sigma_5\left(\frac{n}{12}\right)$
$+\frac{48}{49}\sigma_5\left(\frac{n}{24}\right)+\frac{74}{63}a_1(n)+\frac{1166}{63}a_1\left(\frac{n}{2}\right)+\frac{16\,792}{21}a_1\left(\frac{n}{4}\right)+\frac{29\,696}{7}a_1\left(\frac{n}{8}\right)$

$$+\frac{43}{12}a_2(n)+2a_2\left(\frac{n}{2}\right)+\frac{123}{4}a_2\left(\frac{n}{3}\right)+462a_2\left(\frac{n}{6}\right)-\frac{302}{63}a_3(n)+\frac{16}{9}a_3\left(\frac{n}{2}\right)$$
$$-\frac{4288}{7}a_3\left(\frac{n}{4}\right)+10a_4(n)+54a_4\left(\frac{n}{3}\right)+1632a_5(n)-3360a_6(n)$$
$$+1024a_7(n),$$

$$N(1^5,2,3^3,6^3;n)=-\frac{1}{7056}\sigma_5(n)+\frac{1}{7056}\sigma_5\left(\frac{n}{2}\right)-\frac{3}{784}\sigma_5\left(\frac{n}{3}\right)$$
$$-\frac{1}{1764}\sigma_5\left(\frac{n}{4}\right)+\frac{3}{784}\sigma_5\left(\frac{n}{6}\right)+\frac{16}{441}\sigma_5\left(\frac{n}{8}\right)-\frac{3}{196}\sigma_5\left(\frac{n}{12}\right)+\frac{48}{49}\sigma_5\left(\frac{n}{24}\right)$$
$$-\frac{173}{126}a_1(n)+\frac{173}{21}a_1\left(\frac{n}{2}\right)+\frac{53\,848}{63}a_1\left(\frac{n}{4}\right)+\frac{31\,488}{7}a_1\left(\frac{n}{8}\right)+\frac{16}{3}a_2(n)$$
$$+22a_2\left(\frac{n}{2}\right)-39a_2\left(\frac{n}{3}\right)+330a_2\left(\frac{n}{6}\right)-\frac{254}{63}a_3(n)-\frac{80}{3}a_3\left(\frac{n}{2}\right)$$
$$-\frac{39\,040}{63}a_3\left(\frac{n}{4}\right)+10a_4(n)+162a_4\left(\frac{n}{3}\right)+1792a_5(n)-4224a_6(n)$$
$$+768a_7(n),$$

$$N(1^5,2,3^5,6;n)=-\frac{1}{3528}\sigma_5(n)+\frac{1}{3528}\sigma_5\left(\frac{n}{2}\right)-\frac{3}{392}\sigma_5\left(\frac{n}{3}\right)$$
$$-\frac{1}{1764}\sigma_5\left(\frac{n}{4}\right)+\frac{3}{392}\sigma_5\left(\frac{n}{6}\right)+\frac{16}{441}\sigma_5\left(\frac{n}{8}\right)-\frac{3}{196}\sigma_5\left(\frac{n}{12}\right)+\frac{48}{49}\sigma_5\left(\frac{n}{24}\right)$$
$$-\frac{95}{21}a_1(n)-\frac{1300}{63}a_1\left(\frac{n}{2}\right)+\frac{45\,392}{63}a_1\left(\frac{n}{4}\right)+\frac{29\,696}{7}a_1\left(\frac{n}{8}\right)+6a_2(n)$$
$$+50a_2\left(\frac{n}{2}\right)-126a_2\left(\frac{n}{3}\right)+150a_2\left(\frac{n}{6}\right)-\frac{34}{21}a_3(n)-\frac{488}{9}a_3\left(\frac{n}{2}\right)$$
$$-\frac{35\,456}{63}a_3\left(\frac{n}{4}\right)+10a_4(n)+270a_4\left(\frac{n}{3}\right)+1280a_5(n)-4608a_6(n)$$
$$+256a_7(n),$$

$$N(1^5,2^3,3,6^3;n)=-\frac{41}{183\,456}\sigma_5(n)+\frac{41}{183\,456}\sigma_5\left(\frac{n}{2}\right)+\frac{45}{20\,384}\sigma_5\left(\frac{n}{3}\right)$$
$$+\frac{41}{22\,932}\sigma_5\left(\frac{n}{4}\right)-\frac{45}{20\,384}\sigma_5\left(\frac{n}{6}\right)-\frac{656}{5733}\sigma_5\left(\frac{n}{8}\right)-\frac{45}{2548}\sigma_5\left(\frac{n}{12}\right)$$
$$+\frac{720}{637}\sigma_5\left(\frac{n}{24}\right)+\frac{6268}{819}a_1(n)+\frac{52\,258}{819}a_1\left(\frac{n}{2}\right)+\frac{94\,488}{91}a_1\left(\frac{n}{4}\right)$$
$$+\frac{449\,024}{91}a_1\left(\frac{n}{8}\right)+\frac{13}{4}a_2(n)-50a_2\left(\frac{n}{2}\right)+\frac{831}{4}a_2\left(\frac{n}{3}\right)+978a_2\left(\frac{n}{6}\right)$$
$$-\frac{694}{63}a_3(n)+\frac{608}{9}a_3\left(\frac{n}{2}\right)-\frac{5568}{7}a_3\left(\frac{n}{4}\right)+10a_4(n)-162a_4\left(\frac{n}{3}\right)$$
$$+2784a_5(n)-2976a_6(n)+2048a_7(n),$$
$$N(1^5,2^3,3^3,6;n)=-\frac{41}{91\,728}\sigma_5(n)+\frac{41}{91\,728}\sigma_5\left(\frac{n}{2}\right)+\frac{45}{10\,192}\sigma_5\left(\frac{n}{3}\right)$$
$$+\frac{41}{22\,932}\sigma_5\left(\frac{n}{4}\right)-\frac{45}{10\,192}\sigma_5\left(\frac{n}{6}\right)-\frac{656}{5733}\sigma_5\left(\frac{n}{8}\right)-\frac{45}{2548}\sigma_5\left(\frac{n}{12}\right)$$
$$+\frac{720}{637}\sigma_5\left(\frac{n}{24}\right)+\frac{23\,525}{1638}a_1(n)+\frac{74\,941}{819}a_1\left(\frac{n}{2}\right)+\frac{75\,560}{91}a_1\left(\frac{n}{4}\right)$$
$$+\frac{379\,136}{91}a_1\left(\frac{n}{8}\right)-\frac{5}{3}a_2(n)-110a_2\left(\frac{n}{2}\right)+390a_2\left(\frac{n}{3}\right)+1374a_2\left(\frac{n}{6}\right)$$
$$-\frac{814}{63}a_3(n)+\frac{1280}{9}a_3\left(\frac{n}{2}\right)-\frac{16\,256}{21}a_3\left(\frac{n}{4}\right)+10a_4(n)-486a_4\left(\frac{n}{3}\right)$$
$$+2304a_5(n)-384a_6(n)+2816a_7(n),$$
$$N(1^5,2^5,3,6;n)=-\frac{121}{183\,456}\sigma_5(n)+\frac{121}{183\,456}\sigma_5\left(\frac{n}{2}\right)-\frac{27}{20\,384}\sigma_5\left(\frac{n}{3}\right)$$
$$-\frac{121}{22\,932}\sigma_5\left(\frac{n}{4}\right)+\frac{27}{20\,384}\sigma_5\left(\frac{n}{6}\right)+\frac{1936}{5733}\sigma_5\left(\frac{n}{12}\right)-\frac{27}{2548}\sigma_5\left(\frac{n}{24}\right)$$
$$+\frac{432}{637}a_1(n)-\frac{794}{91}a_1\left(\frac{n}{2}\right)-\frac{1130}{91}a_1\left(\frac{n}{4}\right)+\frac{90\,472}{91}a_1\left(\frac{n}{8}\right)+\frac{446\,464}{91}a_2(n)$$
$$+\frac{37}{4}a_2\left(\frac{n}{2}\right)+74a_2\left(\frac{n}{3}\right)-\frac{945}{4}a_2\left(\frac{n}{6}\right)-378a_3(n)-\frac{6}{7}a_3\left(\frac{n}{2}\right)$$
$$-112a_3\left(\frac{n}{4}\right)-\frac{3520}{7}+10a_4(n)+486a_4\left(\frac{n}{3}\right)+2016a_5(n)-6048a_6(n),$$
$$N(1^6,6^6;n)=-\frac{1}{13\,104}\sigma_5(n)+\frac{1}{13\,104}\sigma_5\left(\frac{n}{2}\right)+\frac{3}{1456}\sigma_5\left(\frac{n}{3}\right)$$
$$+\frac{1}{1638}\sigma_5\left(\frac{n}{4}\right)-\frac{3}{1456}\sigma_5\left(\frac{n}{6}\right)-\frac{32}{819}\sigma_5\left(\frac{n}{8}\right)-\frac{3}{182}\sigma_5\left(\frac{n}{12}\right)$$
$$+\frac{96}{91}\sigma_5\left(\frac{n}{24}\right)+\frac{418}{117}a_1(n)+\frac{1672}{39}a_1\left(\frac{n}{2}\right)+\frac{117\,920}{117}a_1\left(\frac{n}{4}\right)$$
$$+\frac{202\,240}{39}a_1\left(\frac{n}{8}\right)+\frac{13}{6}a_2(n)-\frac{44}{3}a_2\left(\frac{n}{2}\right)+\frac{195}{2}a_2\left(\frac{n}{3}\right)+444a_2\left(\frac{n}{6}\right)$$
$$-\frac{52}{9}a_3(n)+\frac{64}{3}a_3\left(\frac{n}{2}\right)-\frac{6272}{9}a_3\left(\frac{n}{4}\right)+12a_4(n)+1856a_5(n)$$
$$-3264a_6(n)+1536a_7(n),$$
$$N(1^6,3^2,6^4;n)=-\frac{1}{6552}\sigma_5(n)+\frac{1}{6552}\sigma_5\left(\frac{n}{2}\right)+\frac{3}{728}\sigma_5\left(\frac{n}{3}\right)$$
$$+\frac{1}{1638}\sigma_5\left(\frac{n}{4}\right)-\frac{3}{728}\sigma_5\left(\frac{n}{6}\right)-\frac{32}{819}\sigma_5\left(\frac{n}{8}\right)-\frac{3}{182}\sigma_5\left(\frac{n}{12}\right)$$
$$+\frac{96}{91}\sigma_5\left(\frac{n}{24}\right)+\frac{758}{117}a_1(n)+\frac{2356}{39}a_1\left(\frac{n}{2}\right)+\frac{117\,088}{117}a_1\left(\frac{n}{4}\right)$$
$$+\frac{155\,648}{39}a_1\left(\frac{n}{8}\right)+\frac{7}{3}a_2(n)-\frac{100}{3}a_2\left(\frac{n}{2}\right)+177a_2\left(\frac{n}{3}\right)$$
$$+996a_2\left(\frac{n}{6}\right)-\frac{80}{9}a_3(n)+48a_3\left(\frac{n}{2}\right)-\frac{6400}{9}a_3\left(\frac{n}{4}\right)+12a_4(n)$$
$$-108a_4\left(\frac{n}{3}\right)+2560a_5(n)-3072a_6(n)+1536a_7(n),$$

$$N(1^6,3^4,6^2;n)=-\frac{1}{3276}\sigma_5(n)+\frac{1}{3276}\sigma_5\left(\frac{n}{2}\right)+\frac{3}{364}\sigma_5\left(\frac{n}{3}\right)$$
$$+\frac{1}{1638}\sigma_5\left(\frac{n}{4}\right)-\frac{3}{364}\sigma_5\left(\frac{n}{6}\right)-\frac{32}{819}\sigma_5\left(\frac{n}{8}\right)-\frac{3}{182}\sigma_5\left(\frac{n}{12}\right)$$
$$+\frac{96}{91}\sigma_5\left(\frac{n}{24}\right)+\frac{1022}{117}a_1(n)+\frac{1120}{13}a_1\left(\frac{n}{2}\right)+\frac{132\,064}{117}a_1\left(\frac{n}{4}\right)$$

$$+\tfrac{155\,648}{39}a_1\left(\tfrac{n}{8}\right)+\tfrac{4}{3}a_2\left(n\right)-\tfrac{100}{3}a_2\left(\tfrac{n}{2}\right)+132a_2\left(\tfrac{n}{3}\right)$$
$$+996a_2\left(\tfrac{n}{6}\right)-\tfrac{92}{9}a_3\left(n\right)+\tfrac{112}{3}a_3\left(\tfrac{n}{2}\right)-\tfrac{6400}{9}a_3\left(\tfrac{n}{4}\right)$$
$$+12a_4\left(n\right)-108a_4\left(\tfrac{n}{3}\right)+2560a_5\left(n\right)-3072a_6\left(n\right)+1536a_7\left(n\right),$$
$$N(1^6,3^6;n)=-\tfrac{1}{1638}\sigma_5\left(n\right)+\tfrac{1}{819}\sigma_5\left(\tfrac{n}{2}\right)+\tfrac{3}{182}\sigma_5\left(\tfrac{n}{3}\right)-\tfrac{32}{819}\sigma_5\left(\tfrac{n}{4}\right)$$
$$-\tfrac{3}{91}\sigma_5\left(\tfrac{n}{6}\right)+\tfrac{96}{91}\sigma_5\left(\tfrac{n}{12}\right)+\tfrac{152}{13}a_1\left(n\right)+\tfrac{1568}{13}a_1\left(\tfrac{n}{2}\right)+\tfrac{9728}{13}a_1\left(\tfrac{n}{4}\right),$$
$$N(1^6,2^2,6^4;n)=-\tfrac{5}{22\,932}\sigma_5\left(n\right)+\tfrac{5}{22\,932}\sigma_5\left(\tfrac{n}{2}\right)-\tfrac{9}{5096}\sigma_5\left(\tfrac{n}{3}\right)$$
$$-\tfrac{10}{5733}\sigma_5\left(\tfrac{n}{4}\right)+\tfrac{9}{5096}\sigma_5\left(\tfrac{n}{6}\right)+\tfrac{640}{5733}\sigma_5\left(\tfrac{n}{8}\right)-\tfrac{9}{637}\sigma_5\left(\tfrac{n}{12}\right)$$
$$+\tfrac{576}{637}\sigma_5\left(\tfrac{n}{24}\right)+\tfrac{4850}{819}a_1\left(n\right)+\tfrac{34\,124}{819}a_1\left(\tfrac{n}{2}\right)+\tfrac{83\,744}{91}a_1\left(\tfrac{n}{4}\right)$$
$$+\tfrac{284\,672}{91}a_1\left(\tfrac{n}{8}\right)+\tfrac{20}{3}a_2\left(n\right)-16a_2\left(\tfrac{n}{2}\right)+159a_2\left(\tfrac{n}{3}\right)+1128a_2\left(\tfrac{n}{6}\right)$$
$$-\tfrac{800}{63}a_3\left(n\right)+\tfrac{544}{9}a_3\left(\tfrac{n}{2}\right)-\tfrac{14\,080}{21}a_3\left(\tfrac{n}{4}\right)+12a_4\left(n\right)+3456a_5\left(n\right)$$
$$-4224a_6\left(n\right)+1024a_7\left(n\right),$$

$$N(1^6,2^2,3^2,6^2;n)=-\tfrac{5}{11\,466}\sigma_5\left(n\right)+\tfrac{5}{11\,466}\sigma_5\left(\tfrac{n}{2}\right)-\tfrac{9}{2548}\sigma_5\left(\tfrac{n}{3}\right)$$
$$-\tfrac{10}{5733}\sigma_5\left(\tfrac{n}{4}\right)+\tfrac{9}{2548}\sigma_5\left(\tfrac{n}{6}\right)+\tfrac{640}{5733}\sigma_5\left(\tfrac{n}{8}\right)-\tfrac{9}{637}\sigma_5\left(\tfrac{n}{12}\right)$$
$$+\tfrac{576}{637}\sigma_5\left(\tfrac{n}{24}\right)-\tfrac{2312}{819}a_1\left(n\right)+\tfrac{11\,464}{819}a_1\left(\tfrac{n}{2}\right)+\tfrac{306\,560}{273}a_1\left(\tfrac{n}{4}\right)$$
$$+\tfrac{1\,319\,936}{273}a_1\left(\tfrac{n}{8}\right)+\tfrac{28}{3}a_2\left(n\right)+\tfrac{128}{3}a_2\left(\tfrac{n}{2}\right)-78a_2\left(\tfrac{n}{3}\right)+312a_2\left(\tfrac{n}{6}\right)$$
$$-\tfrac{424}{63}a_3\left(n\right)-\tfrac{400}{9}a_3\left(\tfrac{n}{2}\right)-\tfrac{14\,080}{21}a_3\left(\tfrac{n}{4}\right)+12a_4\left(n\right)$$
$$+324a_4\left(\tfrac{n}{3}\right)+3072a_5\left(n\right)-6144a_6\left(n\right)+512a_7\left(n\right),$$
$$N(1^6,2^2,3^4;n)=-\tfrac{5}{5733}\sigma_5\left(n\right)+\tfrac{5}{5733}\sigma_5\left(\tfrac{n}{2}\right)-\tfrac{9}{1274}\sigma_5\left(\tfrac{n}{3}\right)$$

$$-\tfrac{10}{5733}\sigma_5\left(\tfrac{n}{4}\right)+\tfrac{9}{1274}\sigma_5\left(\tfrac{n}{6}\right)+\tfrac{640}{5733}\sigma_5\left(\tfrac{n}{8}\right)-\tfrac{9}{637}\sigma_5\left(\tfrac{n}{12}\right)$$
$$+\tfrac{576}{637}\sigma_5\left(\tfrac{n}{24}\right)-\tfrac{10\,448}{819}a_1\left(n\right)-\tfrac{54\,968}{819}a_1\left(\tfrac{n}{2}\right)+\tfrac{201\,728}{273}a_1\left(\tfrac{n}{4}\right)$$
$$+\tfrac{1\,226\,752}{273}a_1\left(\tfrac{n}{8}\right)+\tfrac{28}{3}a_2\left(n\right)+\tfrac{400}{3}a_2\left(\tfrac{n}{2}\right)-348a_2\left(\tfrac{n}{3}\right)$$
$$-696a_2\left(\tfrac{n}{6}\right)+\tfrac{440}{63}a_3\left(n\right)-\tfrac{1312}{9}a_3\left(\tfrac{n}{2}\right)-\tfrac{8704}{21}a_3\left(\tfrac{n}{4}\right)$$
$$+8a_4\left(n\right)+648a_4\left(\tfrac{n}{3}\right)-6144a_6\left(n\right)-1024a_7\left(n\right),$$
$$N(1^6,2^4,0,6^2;n)=-\tfrac{61}{91\,728}\sigma_5\left(n\right)+\tfrac{61}{91\,728}\sigma_5\left(\tfrac{n}{2}\right)$$
$$+\tfrac{27}{10\,192}\sigma_5\left(\tfrac{n}{3}\right)+\tfrac{61}{11\,466}\sigma_5\left(\tfrac{n}{4}\right)-\tfrac{27}{10\,192}\sigma_5\left(\tfrac{n}{6}\right)$$
$$-\tfrac{1952}{5733}\sigma_5\left(\tfrac{n}{8}\right)-\tfrac{27}{1274}\sigma_5\left(\tfrac{n}{12}\right)+\tfrac{864}{637}\sigma_5\left(\tfrac{n}{24}\right)$$
$$+\tfrac{5192}{819}a_1\left(n\right)+\tfrac{75\,668}{819}a_1\left(\tfrac{n}{2}\right)+\tfrac{465\,664}{273}a_1\left(\tfrac{n}{4}\right)$$
$$+\tfrac{2\,073\,088}{273}a_1\left(\tfrac{n}{8}\right)+\tfrac{17}{2}a_2\left(n\right)-\tfrac{116}{3}a_2\left(\tfrac{n}{2}\right)+\tfrac{345}{2}a_2\left(\tfrac{n}{3}\right)$$
$$+948a_2\left(\tfrac{n}{6}\right)-\tfrac{956}{63}a_3\left(n\right)+\tfrac{352}{9}a_3\left(\tfrac{n}{2}\right)-\tfrac{22\,912}{21}a_3\left(\tfrac{n}{4}\right)$$
$$+12a_4\left(n\right)+4800a_5\left(n\right)-5952a_6\left(n\right)+2560a_7\left(n\right),$$

$$N(1^6,2^4,3^2;n)=-\tfrac{61}{45\,864}\sigma_5\left(n\right)+\tfrac{61}{45\,864}\sigma_5\left(\tfrac{n}{2}\right)+\tfrac{27}{5096}\sigma_5\left(\tfrac{n}{3}\right)$$
$$+\tfrac{61}{11\,466}\sigma_5\left(\tfrac{n}{4}\right)-\tfrac{27}{5096}\sigma_5\left(\tfrac{n}{6}\right)-\tfrac{1952}{5733}\sigma_5\left(\tfrac{n}{8}\right)-\tfrac{27}{1274}\sigma_5\left(\tfrac{n}{12}\right)$$
$$+\tfrac{864}{637}\sigma_5\left(\tfrac{n}{24}\right)+\tfrac{20\,758}{819}a_1\left(n\right)+\tfrac{91\,276}{819}a_1\left(\tfrac{n}{2}\right)+\tfrac{113\,312}{273}a_1\left(\tfrac{n}{4}\right)$$
$$+\tfrac{628\,736}{273}a_1\left(\tfrac{n}{8}\right)-9a_2\left(n\right)-\tfrac{556}{3}a_2\left(\tfrac{n}{2}\right)+687a_2\left(\tfrac{n}{3}\right)$$
$$+2364a_2\left(\tfrac{n}{6}\right)-\tfrac{1072}{63}a_3\left(n\right)+\tfrac{2480}{9}a_3\left(\tfrac{n}{2}\right)-\tfrac{16\,640}{21}a_3\left(\tfrac{n}{4}\right)$$
$$+12a_4\left(n\right)-972a_4\left(\tfrac{n}{3}\right)+1536a_5\left(n\right)+3072a_6\left(n\right)+3584a_7\left(n\right),$$
$$N(1^6,2^6;n)=-\tfrac{1}{504}\sigma_5\left(n\right)+\tfrac{1}{504}\sigma_5\left(\tfrac{n}{2}\right)-\tfrac{1}{63}\sigma_5\left(\tfrac{n}{4}\right)$$

$$+\tfrac{64}{63}\sigma_5\left(\tfrac{n}{8}\right)-a_2\left(n\right)-8a_2\left(\tfrac{n}{2}\right)+12a_4\left(n\right),$$
$$N(1^7,2,3,6^3;n)=-\tfrac{41}{91\,728}\sigma_5\left(n\right)+\tfrac{41}{91\,728}\sigma_5\left(\tfrac{n}{2}\right)+\tfrac{45}{10\,192}\sigma_5\left(\tfrac{n}{3}\right)$$
$$+\tfrac{41}{22\,932}\sigma_5\left(\tfrac{n}{4}\right)-\tfrac{45}{10\,192}\sigma_5\left(\tfrac{n}{6}\right)-\tfrac{656}{5733}\sigma_5\left(\tfrac{n}{8}\right)-\tfrac{45}{2548}\sigma_5\left(\tfrac{n}{12}\right)$$
$$+\tfrac{720}{637}\sigma_5\left(\tfrac{n}{24}\right)+\tfrac{6689}{546}a_1\left(n\right)+\tfrac{27\,589}{273}a_1\left(\tfrac{n}{2}\right)+\tfrac{129\,432}{91}a_1\left(\tfrac{n}{4}\right)$$
$$+\tfrac{1\,370\,368}{273}a_1\left(\tfrac{n}{8}\right)+\tfrac{20}{3}a_2\left(n\right)-\tfrac{194}{3}a_2\left(\tfrac{n}{2}\right)+333a_2\left(\tfrac{n}{3}\right)$$
$$+1494a_2\left(\tfrac{n}{6}\right)-\tfrac{134}{7}a_3\left(n\right)+\tfrac{352}{3}a_3\left(\tfrac{n}{2}\right)-\tfrac{19\,840}{21}a_3\left(\tfrac{n}{4}\right)$$
$$+14a_4\left(n\right)-162a_4\left(\tfrac{n}{3}\right)+4992a_5\left(n\right)-4608a_6\left(n\right)+2304a_7\left(n\right),$$
$$N(1^7,2,3^3,6;n)=-\tfrac{41}{45\,864}\sigma_5\left(n\right)+\tfrac{41}{45\,864}\sigma_5\left(\tfrac{n}{2}\right)+\tfrac{45}{5096}\sigma_5\left(\tfrac{n}{3}\right)$$
$$+\tfrac{41}{22\,932}\sigma_5\left(\tfrac{n}{4}\right)-\tfrac{45}{5096}\sigma_5\left(\tfrac{n}{6}\right)-\tfrac{656}{5733}\sigma_5\left(\tfrac{n}{8}\right)-\tfrac{45}{2548}\sigma_5\left(\tfrac{n}{12}\right)$$
$$+\tfrac{720}{637}\sigma_5\left(\tfrac{n}{24}\right)+\tfrac{14\,425}{819}a_1\left(n\right)+\tfrac{98\,740}{819}a_1\left(\tfrac{n}{2}\right)+\tfrac{101\,040}{91}a_1\left(\tfrac{n}{4}\right)$$

$$+\frac{1020\,928}{273}a_1\left(\frac{n}{8}\right)-\frac{350}{3}a_2\left(\frac{n}{2}\right)+480a_2\left(\frac{n}{3}\right)+1842a_2\left(\frac{n}{6}\right)$$
$$-\frac{1138}{63}a_3(n)+\frac{1544}{9}a_3\left(\frac{n}{2}\right)-\frac{6016}{7}a_3\left(\frac{n}{4}\right)+14a_4(n)$$
$$-486a_4\left(\frac{n}{3}\right)+3840a_5(n)-1536a_6(n)+2816a_7(n),$$
$$N(1^7,2^3,3,6;n)=-\frac{121}{91\,728}\sigma_5(n)+\frac{121}{91\,728}\sigma_5\left(\frac{n}{2}\right)-\frac{27}{10\,192}\sigma_5\left(\frac{n}{3}\right)$$
$$-\frac{121}{22\,932}\sigma_5\left(\frac{n}{4}\right)+\frac{27}{10\,192}\sigma_5\left(\frac{n}{6}\right)+\frac{1936}{5733}\sigma_5\left(\frac{n}{8}\right)-\frac{27}{2548}\sigma_5\left(\frac{n}{12}\right)$$
$$+\frac{432}{637}\sigma_5\left(\frac{n}{24}\right)-\frac{8291}{1638}a_1(n)+\frac{17\,789}{819}a_1\left(\frac{n}{2}\right)+\frac{364\,600}{273}a_1\left(\frac{n}{4}\right)$$
$$+\frac{1362\,688}{273}a_1\left(\frac{n}{8}\right)+13a_2(n)+\frac{178}{3}a_2\left(\frac{n}{2}\right)-138a_2\left(\frac{n}{3}\right)$$
$$+138a_2\left(\frac{n}{6}\right)-\frac{542}{63}a_3(n)-\frac{656}{9}a_3\left(\frac{n}{2}\right)-\frac{13\,696}{21}a_3\left(\frac{n}{4}\right)$$
$$+14a_4(n)+486a_4\left(\frac{n}{3}\right)+4224a_5(n)-7680a_6(n)+256a_7(n),$$

$$N(1^8,6^4;n)=-\frac{5}{11\,466}\sigma_5(n)+\frac{5}{11\,466}\sigma_5\left(\frac{n}{2}\right)-\frac{9}{2548}\sigma_5\left(\frac{n}{3}\right)$$
$$-\frac{10}{5733}\sigma_5\left(\frac{n}{4}\right)+\frac{9}{2548}\sigma_5\left(\frac{n}{6}\right)+\frac{640}{5733}\sigma_5\left(\frac{n}{8}\right)-\frac{9}{637}\sigma_5\left(\frac{n}{12}\right)$$
$$+\frac{576}{637}\sigma_5\left(\frac{n}{24}\right)+\frac{10\,792}{819}a_1(n)+\frac{66\,064}{819}a_1\left(\frac{n}{2}\right)+\frac{388\,096}{273}a_1\left(\frac{n}{4}\right)$$
$$+\frac{471\,040}{91}a_1\left(\frac{n}{8}\right)+12a_2(n)-48a_2\left(\frac{n}{2}\right)+354a_2\left(\frac{n}{3}\right)+1320a_2\left(\frac{n}{6}\right)$$
$$-\frac{1600}{63}a_3(n)+\frac{1472}{9}a_3\left(\frac{n}{2}\right)-\frac{19\,456}{21}a_3\left(\frac{n}{4}\right)+16a_4(n)+6144a_5(n)$$
$$-6144a_6(n)+2048a_7(n),$$

$$N(1^8,3^2,6^2;n)=-\frac{5}{5733}\sigma_5(n)+\frac{5}{5733}\sigma_5\left(\frac{n}{2}\right)-\frac{9}{1274}\sigma_5\left(\frac{n}{3}\right)$$
$$-\frac{10}{5733}\sigma_5\left(\frac{n}{4}\right)+\frac{9}{1274}\sigma_5\left(\frac{n}{6}\right)+\frac{640}{5733}\sigma_5\left(\frac{n}{8}\right)-\frac{9}{637}\sigma_5\left(\frac{n}{12}\right)$$
$$+\frac{576}{637}\sigma_5\left(\frac{n}{24}\right)+\frac{2656}{819}a_1(n)+\frac{25\,840}{819}a_1\left(\frac{n}{2}\right)+\frac{341\,504}{273}a_1\left(\frac{n}{4}\right)$$
$$+\frac{471\,040}{91}a_1\left(\frac{n}{8}\right)+\frac{40}{3}a_2(n)-48a_2\left(\frac{n}{2}\right)+408a_2\left(\frac{n}{3}\right)$$
$$+1320a_2\left(\frac{n}{6}\right)-\frac{1072}{63}a_3(n)+\frac{1088}{9}a_3\left(\frac{n}{2}\right)-\frac{19\,456}{21}a_3\left(\frac{n}{4}\right)$$
$$+16a_4(n)+6144a_5(n)-6144a_6(n)+2048a_7(n),$$
$$N(1^8,3^4;n)=-\frac{10}{5733}\sigma_5(n)-\frac{9}{637}\sigma_5\left(\frac{n}{3}\right)+\frac{640}{5733}\sigma_5\left(\frac{n}{4}\right)$$
$$+\frac{576}{637}\sigma_5\left(\frac{n}{12}\right)-\frac{704}{91}a_1(n)-\frac{2048}{21}a_1\left(\frac{n}{2}\right)-\frac{77\,824}{273}a_1\left(\frac{n}{4}\right)$$
$$+\frac{32}{3}a_2(n)+432a_2\left(\frac{n}{3}\right)+\frac{256}{21}a_3(n)+\frac{2048}{21}a_3\left(\frac{n}{2}\right),$$
$$N(1^8,2^2,6^2;n)=-\frac{61}{45\,864}\sigma_5(n)+\frac{61}{45\,864}\sigma_5\left(\frac{n}{2}\right)+\frac{27}{5096}\sigma_5\left(\frac{n}{3}\right)$$
$$+\frac{61}{11\,466}\sigma_5\left(\frac{n}{4}\right)-\frac{27}{5096}\sigma_5\left(\frac{n}{6}\right)-\frac{1952}{5733}\sigma_5\left(\frac{n}{8}\right)-\frac{27}{1274}\sigma_5\left(\frac{n}{12}\right)$$
$$+\frac{864}{637}\sigma_5\left(\frac{n}{24}\right)+\frac{3704}{273}a_1(n)+\frac{42\,680}{273}a_1\left(\frac{n}{2}\right)+\frac{217\,344}{91}a_1\left(\frac{n}{4}\right)$$
$$+\frac{706\,560}{91}a_1\left(\frac{n}{8}\right)+17a_2(n)-68a_2\left(\frac{n}{2}\right)+369a_2\left(\frac{n}{3}\right)$$
$$+1980a_2\left(\frac{n}{6}\right)-\frac{656}{21}a_3(n)+\frac{352}{3}a_3\left(\frac{n}{2}\right)-\frac{9728}{7}a_3\left(\frac{n}{4}\right)$$
$$+16a_4(n)+9216a_5(n)-9216a_6(n)+3072a_7(n),$$

$$N(1^8,2^2,3^2;n)=-\frac{61}{22\,932}\sigma_5(n)+\frac{61}{22\,932}\sigma_5\left(\frac{n}{2}\right)+\frac{27}{2548}\sigma_5\left(\frac{n}{3}\right)$$
$$+\frac{61}{11\,466}\sigma_5\left(\frac{n}{4}\right)-\frac{27}{2548}\sigma_5\left(\frac{n}{6}\right)-\frac{1952}{5733}\sigma_5\left(\frac{n}{8}\right)-\frac{27}{1274}\sigma_5\left(\frac{n}{12}\right)$$
$$+\frac{864}{637}\sigma_5\left(\frac{n}{24}\right)+\frac{23\,134}{819}a_1(n)+\frac{102\,472}{819}a_1\left(\frac{n}{2}\right)+\frac{148\,256}{273}a_1\left(\frac{n}{4}\right)$$
$$+\frac{628\,736}{273}a_1\left(\frac{n}{8}\right)-8a_2(n)-\frac{556}{3}a_2\left(\frac{n}{2}\right)+768a_2\left(\frac{n}{3}\right)$$
$$+2364a_2\left(\frac{n}{6}\right)-\frac{1108}{63}a_3(n)+\frac{2768}{9}a_3\left(\frac{n}{2}\right)-\frac{16\,640}{21}a_3\left(\frac{n}{4}\right)$$
$$+12a_4(n)-972a_4\left(\frac{n}{3}\right)+1536a_5(n)+3072a_6(n)+3584a_7(n),$$

$$N(1^8,2^4;n)=-\frac{1}{252}\sigma_5(n)+\frac{1}{252}\sigma_5\left(\frac{n}{2}\right)-\frac{1}{63}\sigma_5\left(\frac{n}{4}\right)$$
$$+\frac{64}{63}\sigma_5\left(\frac{n}{8}\right)-2a_2(n)-8a_2\left(\frac{n}{2}\right)+16a_4(n),$$
$$N(1^9,2,3,6;n)=-\frac{121}{45\,864}\sigma_5(n)+\frac{121}{45\,864}\sigma_5\left(\frac{n}{2}\right)-\frac{27}{5096}\sigma_5\left(\frac{n}{3}\right)$$
$$-\frac{121}{22\,932}\sigma_5\left(\frac{n}{4}\right)+\frac{27}{5096}\sigma_5\left(\frac{n}{6}\right)+\frac{1936}{5733}\sigma_5\left(\frac{n}{8}\right)-\frac{27}{2548}\sigma_5\left(\frac{n}{12}\right)$$
$$+\frac{432}{637}\sigma_5\left(\frac{n}{24}\right)+\frac{2083}{819}a_1(n)+\frac{62\,332}{819}a_1\left(\frac{n}{2}\right)+\frac{542\,960}{273}a_1\left(\frac{n}{4}\right)$$
$$+\frac{1805\,312}{273}a_1\left(\frac{n}{8}\right)+22a_2(n)+\frac{62}{3}a_2\left(\frac{n}{2}\right)+66a_2\left(\frac{n}{3}\right)$$
$$+798a_2\left(\frac{n}{26}\right)-\frac{1630}{63}a_3(n)+\frac{248}{9}a_3\left(\frac{n}{2}\right)-\frac{20\,864}{21}a_3\left(\frac{n}{4}\right)$$
$$+18a_4(n)+486a_4\left(\frac{n}{3}\right)+8448a_5(n)-10\,752a_6(n)+1280a_7(n),$$
$$N(1^{10},6^2;n)=-\frac{61}{22\,932}\sigma_5(n)+\frac{61}{22\,932}\sigma_5\left(\frac{n}{2}\right)+\frac{27}{2548}\sigma_5\left(\frac{n}{3}\right)$$
$$+\frac{61}{11\,466}\sigma_5\left(\frac{n}{4}\right)-\frac{27}{2548}\sigma_5\left(\frac{n}{6}\right)-\frac{1952}{5733}\sigma_5\left(\frac{n}{8}\right)-\frac{27}{1274}\sigma_5\left(\frac{n}{12}\right)$$
$$+\frac{864}{637}\sigma_5\left(\frac{n}{24}\right)+\frac{23\,498}{819}a_1(n)+\frac{274\,280}{819}a_1\left(\frac{n}{2}\right)+\frac{1295\,584}{273}a_1\left(\frac{n}{4}\right)$$

$$+\frac{3610\,624}{273}a_1\left(\frac{n}{8}\right)+40a_2\left(n\right)+\frac{148}{3}a_2\left(\frac{n}{2}\right)-192a_2\left(\frac{n}{3}\right)$$
$$+1596a_2\left(\frac{n}{6}\right)-\frac{4412}{63}a_3\left(n\right)+\frac{496}{9}a_3\left(\frac{n}{2}\right)-\frac{41\,728}{21}a_3\left(\frac{n}{4}\right)$$
$$+20a_4\left(n\right)+972a_4\left(\frac{n}{3}\right)+16\,896a_5\left(n\right)-21\,504a_6\left(n\right)+2560a_7\left(n\right),$$
$$N(1^{10},3^2;n)=-\frac{61}{11\,466}\sigma_5\left(n\right)+\frac{61}{5733}\sigma_5\left(\frac{n}{2}\right)+\frac{27}{1274}\sigma_5\left(\frac{n}{3}\right)$$
$$-\frac{1952}{5733}\sigma_5\left(\frac{n}{4}\right)\frac{27}{637}\sigma_5\left(\frac{n}{6}\right)+\frac{864}{637}\sigma_5\left(\frac{n}{12}\right)+\frac{3240}{91}a_1\left(n\right)+\frac{2848}{13}a_1\left(\frac{n}{2}\right)$$
$$+\frac{100\,864}{91}a_1\left(\frac{n}{4}\right)-\frac{128}{7}a_3\left(n\right)+\frac{1024}{7}a_3\left(\frac{n}{2}\right),$$
$$N(1^{10},2^2;n)=-\frac{1}{126}\sigma_5\left(n\right)+\frac{1}{126}\sigma_5\left(\frac{n}{2}\right)-\frac{1}{63}\sigma_5\left(\frac{n}{4}\right)+\frac{64}{63}\sigma_5\left(\frac{n}{8}\right)$$
$$-8a_2\left(\frac{n}{2}\right)+16a_4\left(n\right),$$
$$N(1^{12};n)=-\frac{1}{63}\sigma_5\left(n\right)+\frac{64}{63}\sigma_5\left(\frac{n}{4}\right)+16a_2\left(n\right).$$

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