



# Electrodynamic Forces between Electrical Conductors and Cylindrical Magnetic Shields

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## Authors' contributions

This work was carried out in collaboration between all authors. Author MM designed the study, performed the statistical analysis, wrote the protocol, and wrote the first draft of the manuscript and managed literature searches. Authors ZB and SB managed the analyses of the study and literature searches. All authors read and approved the final manuscript.

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## ABSTRACT

**Aims:** This paper demonstrates a method for calculating electrodynamic stress in single-phase and three-phase isolated conductors. Conductors are coated with a cylindrical shield made out of the material containing a magnetic parameter. Special emphasis is placed on induced eddy current of a shield and its effect on reduction of electrodynamic stress of three-phase conductors.

**Methodology:** The paper starts with the assumption that the cylindrical structure of the conductor shield is infinitely long  $\sigma=0$ . Inside and outside the shield applies Laplace differential equation for the magnetic vector potential [1,3]. Short circuit currents flow through the three eccentric positioned conductors and create magnetic induction or fluxes in a cylindrical shield. AC power corresponds to the time-varying resulting flux that induces eddy currents in the cylinder. Needed values of induction and fluxes relevant to eddy currents and electrodynamic forces can be determined by a method which is based on the calculation of the magnetic vector potentials and Poisson differential equation,  $A=\mu J$  [2,4,5]. This procedure requires the establishment of a large number of boundary conditions and taking into account the superposition of multiphase conductive structure values. The impact

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measure of eddy currents in such an eccentric three-phase structure can be determined by using a similar simple procedure which one of the authors applied in the [6-17].

**Conclusion:** Electrodynamics forces, according to relationship (32), are significantly lower in shields which encompass only one phase of a conductor due to a protective effect of a shield. Eddy currents, as is demonstrated in this paper, significantly reduce magnetic field intensity produced by currents in conductors. Due to this effect, main electrodynamics forces in one-phase structures with shields, don't affect the conductor but only affect the shield.

*Keywords: Magnetic flux; eddy currents; conductor; electro-dynamic forces; enclosure.*

## 1. INTRODUCTION

Magnetic screen application in form of a cylindrical shield creates a condition for controlling magnetic flux losses. Flux losses could be directed to areas where they would have a reduced value or would be used for other purposes. In other words, in this way a possibility is created for influencing some physical sizes such as reduction of electrodynamics forces in extremely disturbed conditions (for example, in a case of a short circuit in transformer winding) [1-3]. Beside this, under normal conditions, winding reactance values can be reduced hence reducing heating of certain parts in transformers and engines [11,13,15-17]. Magnetic shield is made out of electrical steel sheets. Depending on position in relation to the magnetic field line direction they are divided into horizontal and vertical cylindrical shields. This paper discusses vertical cylindrical shields made out of a material with a good magnetic conductivity which are cross placed in relation to the magnetic field lines [4-7].

## 2. THE METODOLOGY ON INVESTIGATING THE FLUENCE OF EDDY CURRENT IN THE CONDUCTOR

In a case of alternating current, magnetic flux in a shield suppresses eddy current distributed in a magnetic shield layer at a certain depth which defines its electric and magnetic properties. Electrodynamics action is followed by a simultaneous increase of strength loss and shield heating.

Magnetic field created by a current through a conductor eq. (1),  $i_1$  affects the second conductor with a current  $i_2$  and depending on current direction results in attracting or repelling electrodynamic forces (see Fig.1.c.). Magnetic field strength is determined using Biot-Savart law [1]:

$$d\vec{H}_1 = \frac{1}{4\pi} \frac{i_1 \cdot d\vec{l}_1 \times \vec{r}}{r^3} \quad (1)$$

Electrodynamic force is determined by Laplace formula, that is

$$d\vec{F} = i_2 \cdot d\vec{l}_2 \times \vec{B}_1 \quad (2)$$

and after replacing eq. (1) with eq. (2) we get a relationship

$$\vec{F} = \frac{\mu}{4\pi} i_1 \cdot i_2 \int_{l_1} \int_{l_2} \frac{dl_1 \cdot dl_2}{r^2} \sin \alpha \quad (3)$$

If we neglect the influence of a restrained component of a short circuit transient current, it can be demonstrated in a relationship [2] :

$$i_{sh-cir} = I_0(1 - \cos \omega t) \quad (4)$$

$i_{short-circuit} = i_{sh-cir}$  - current value of a short circuit current,

$I_0$  - initial value of direct short circuit current,

$t$  - time from the moment of emergence of a short circuit current,

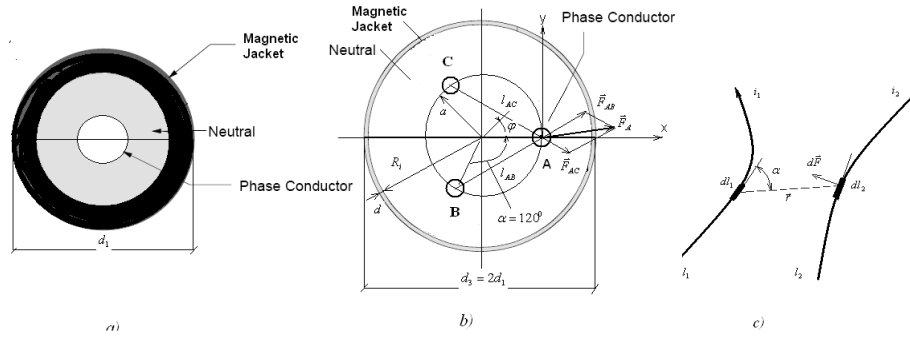
$\omega = 2\pi f$  - cyclic frequency of network voltage.

Magnetic field around a conductor with a short circuit current is calculated according to equation

$$b = B_0(1 - \cos \omega t) \quad (5)$$

Electrodynamic force per unit length of the conductor with a short circuit current in a magnetic field close to a second parallel conductor is calculated in a relationship eq. (2):

$$\begin{aligned} d\vec{F} &= i_2 \cdot d\vec{l}_2 \times \vec{B}_1 = I_0(1 - \cos \omega t) \cdot dl_2 \cdot B_0 \cdot (1 - \cos \omega t) \\ \vec{F} &= \int_0^l i_2 \cdot d\vec{l}_2 \times \vec{B}_1 = l \cdot B_0 \cdot I_0(1 - \cos \omega t)^2 \\ f &= \vec{F} / l = B_0 \cdot I_0(1 - \cos \omega t)^2 \quad (6) \end{aligned}$$



**Fig. 1. Position in a single-pole shield conductor (a), in a three-pole shield (b) and electrodynamic force between two conductors with a current  $i_1$  i  $i_2$**

The relationship eq. (6) we can write as:

$$f = \frac{3}{2} B_0 \cdot I_0 \left( 1 - \frac{4}{3} \cos \omega t + \frac{1}{3} \cos 2\omega t \dots \right) \quad (7)$$

The magnetic induction at a distance from a second line conductor is according to Ampere law:

$$B = \frac{I_0}{2\pi a} \quad (8)$$

If eq. (8) is replaced by eq. (7) and  $I_0 = \sqrt{2} \cdot \dot{I}_{sh-cir}$  [2], in a first half-cycle of a short circuit current, then:

$$f = \frac{6\dot{I}_{sh-cir}^2}{a} \left( 1 - \frac{4}{3} \cos \omega t + \frac{1}{3} \cos 2\omega t \right) \cdot 10^{-7} \quad (9)$$

$\dot{I}_{sh-cir}$  - effective value of short circuit current in a sub-transient period,  $a$  - distance between parallel conductors.

From the relationship eq. (9) it is apparent that beside electrodynamic force which emerged as a result of a direct current component, there exist two forces created by alternating current frequency of (50 Hz), and alternating current frequency of (100 Hz). With firmly positioned conductors the highest value emerges when condition to add all three components together is created, that is, at the moment for  $za$  ( $\omega t = \pi$ ),  $(1 + 4/3 + 1/3) = 7/3$ .

In case of a strongest disturbance – three phase short circuit, the maximum force is exerted on the middle conductor, in line conductors' arrangement [2], and calculation is reformed

under the assumption that currents in phases B and C have the same direction, and then:

$$f_3 = 2 \frac{\dot{I}_{A,sh-cir}}{a} (\dot{I}_{B,sh-cir} - \dot{I}_{C,sh-cir}) \cdot 10^{-7} \quad (10)$$

A conductor of circular cross section lies on a cylinder axis with a magnetic parameter  $\mu$ , placed in a homogeneous magnetic field ( $H_0$ ), Fig. 2. With dimensions of: Inside and outside diameter of the cylindrical shell ( $D=2 R_0$ ,  $D_f=2 R_f$ ), thickness of cylindrical shell ( $d$ ) Fig. 2a.

The first assumption is that the structure of the cylindrical shell is infinitely long, that a shield doesn't contain a conductor – space without a current ( $\sigma=0$ ). For inside and outside shield Laplace differential equation for magnetic vector potential is applied ( $\Delta \vec{A}=0$ ), [3]: In the region where the current density equals zero, a simple method for determining the scalar potential function can be applied. ( $rot \vec{H} \neq 0$ ) only in the current occupied area which is why it is called eddy current space. Outside eddy current space ( $rot \vec{H} = 0$ ), and the vector field  $H$  is considered as a potential. In this area we have ( $\vec{H} = -grad \varphi_m$ ), where ( $\varphi_m$ ), is a scalar function called the scalar potential [1-3].

A cylinder with a magnetic parameter  $\mu$  subdued to magnetic polarisation creates its own magnetic field, and an arbitrary point field ( $M(r,\theta)$ ) of cylindric coordinates in the neighbouring space, outside the shield determines the magnetic scalar potential ( $\varphi_{m0}$ ) that is:

$$(H_0 = -grad \varphi_{m0} = -\frac{\partial \varphi_{m0}}{\partial y}), \text{ where is scalar}$$

potential:  $\varphi_{m0} = -H_0 y + C_{Int}$ .

Under the assumption that on the cylinder axis ( $y=0$ ) the potential is ( $\varphi_{m0}=0$ ) and by inserting ( $y=r \cos\theta$ ) the potential is ( $\varphi_{m0}=-H_0 r \cos\theta$ ). Furthermore, the scalar potential, the potential of the fictive magnetic dipole with positive and negative magnetic load line at a distance  $d$ , which substitutes the influence of homogeneous magnetic cylinder, in exterior point  $M$  result potential, is derived (second coefficient is proportional to  $1/r$  according to [3] ), where:  $r$  - distance from the shield axis,  $\theta$  - rotation angle (polar coordinates)

$$\varphi_{my} = -H_0 r \cdot \cos\theta + \frac{C_{Int.1}}{r} \cos\theta \quad (11)$$

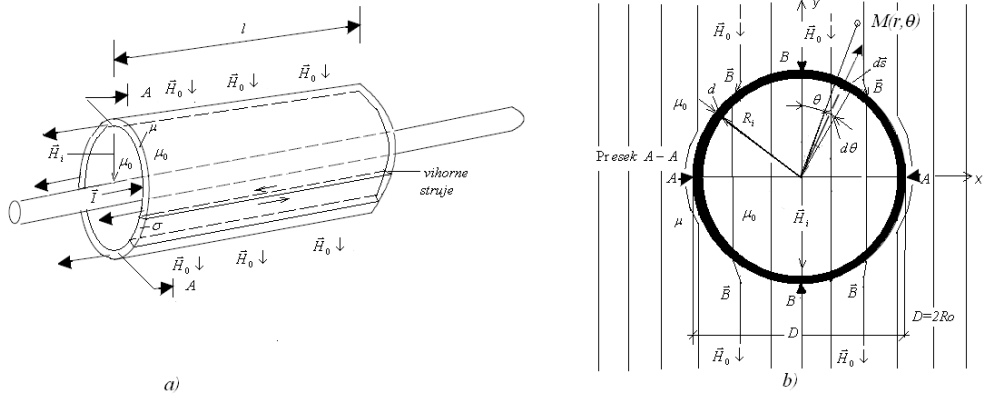
and  $C_{Int.1}$  is an integration constant proportional to the moment of the fictive magnetic dipole in relation to the exterior surface area of a cylinder eq. (3). Interior field of a cylinder which is placed in an unknown homogeneous magnetic field is also the homogeneous. In a shield, a field

consists of unknown homogeneous field component and a fictive dipole field placed on the cylinder axis. The potential of points in the very shield is determined in a relationship:

$$\varphi_{mFe} = -H_{Fe} r \cdot \cos\theta + \frac{C_{Int.2}}{r} \cos\theta \quad (11a)$$

Here ( $H_{Fe}$ ) is an unknown homogeneous field value in a shield in case unknown field doesn't exist ( $H_0$ ), ( $C_{Int.2}$ ) is an integration constant proportional to the moment of fictive magnetic dipole in relation to the interior surface area of a cylinder [1,3]. And finally, the field ( $H_i$ ) between the axis and interior surface area of a cylinder is also homogeneous and emerges as a result of field presence attained through polarisation in relation to both surface areas of a cylinder: exterior and interior. The potential point inside the screen space equals

$$\varphi_{m.i} = -H_i r \cdot \cos\theta \quad (11b)$$



**Fig. 2. Current through a conductor and returning current distribution in a shield with outstanding magnetic characteristics**

The constants in prior relationships, for ( $rot\vec{H}=0$ ), are determined based on tangent vector component equation principle ( $\vec{H}$ ) and normal vector components ( $\vec{B}$ ), on bordering surface areas of a cylinder:

$$\text{interior: } (r = R_i), \Leftrightarrow \left(-\frac{1}{r} \frac{\partial \varphi_{mi}}{\partial r}\right)_{r=R_i} = \left(-\frac{1}{r} \frac{\partial \varphi_{mFe}}{\partial r}\right)_{r=R_i}, \quad (12)$$

$$\mu_0 \left(-\frac{1}{r} \frac{\partial \varphi_{mi}}{\partial r}\right)_{r=R_i} = \mu \left(-\frac{1}{r} \frac{\partial \varphi_{mFe}}{\partial r}\right)_{r=R_i}, \Leftrightarrow \mu_0 H_i = \mu_{Fe} \left(H_{Fe0} + \frac{2C_{Int.2}}{R_i^2}\right), \quad (12a)$$

$$\text{exterior: } (r = R_o), \left(-\frac{1}{r} \frac{\partial \varphi_{mFe}}{\partial r}\right)_{r=R_o} = \left(-\frac{1}{r} \frac{\partial \varphi_{my}}{\partial r}\right)_{r=R_o}, \Leftrightarrow H_{Fe} - \frac{C_{Int.2}}{R_o^2} = H_0 + \frac{C_{Int.1}}{R_o^2}, \quad (13)$$

$$\text{and } \mu \left(-\frac{\partial \varphi_{mFe}}{\partial r}\right)_{r=R_o} = \mu_0 \left(-\frac{\partial \varphi_{my}}{\partial r}\right)_{r=R_o} \Leftrightarrow \mu \left(H_{Fe} + \frac{2C_{Int.2}}{R_o^2}\right) = \mu_0 \left(H_0 + \frac{2C_{Int.1}}{R_o^2}\right) \quad (13.a)$$

The constants ( $H_{Fe}$ ) and ( $C_{Int2}$ ) are determined based on two conditions eq. 12 and eq. 12.a (interior surface area of a cylinder):

$$H_{Fe} = (2 + \frac{\mu_0}{\mu}) \frac{H_i}{3} \text{ and } C_{Int.2} = -R_i^2 (1 - \frac{\mu_0}{\mu}) \frac{H_i}{3} \quad (13.b)$$

When these two relationships are substituted into two conditions (eq. 13 and eq. 13a) an equation for calculating magnetic field inside a cylinder screen is derived. The cylindric shield results from a cross-section of two concentric cylinders with radius of ( $r=R_0$ ) and ( $r=R_i$ ). After substitution and sorting it out, an equation for magnetic field strength is derived ( $H_i$ ) inside a cylinder screen which is placed in ( $H_0$ ) unknown homogeneous magnetic field:

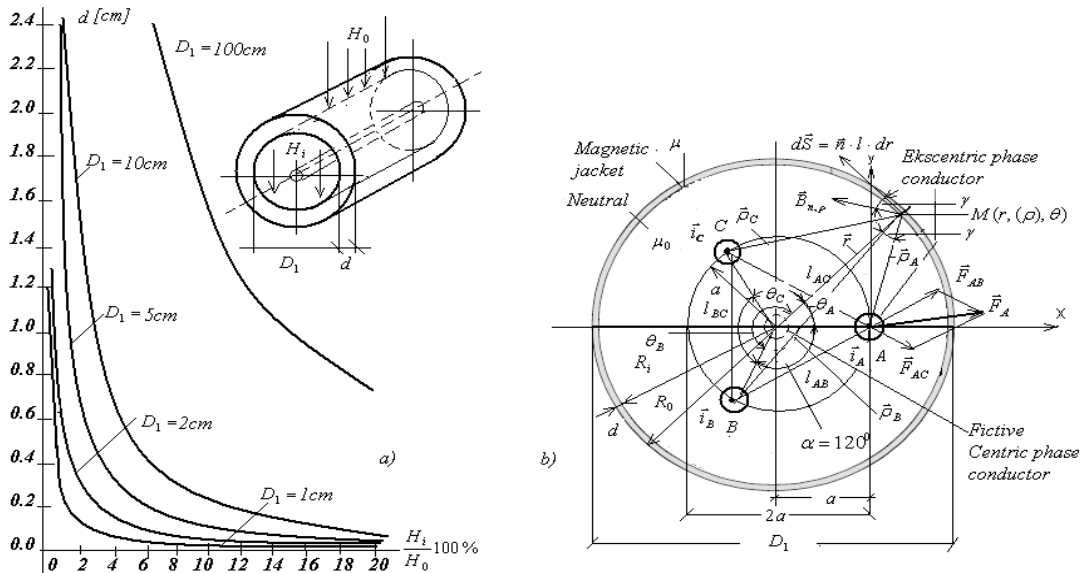
$$H_i = \frac{H_0}{1 + \frac{1}{4} (1 - \frac{R_i^2}{R_0^2}) (\frac{\mu_0}{\mu} + \frac{\mu}{\mu_0} - 2)} \approx \frac{H_0}{1 + \frac{1}{4} (1 - \frac{R_i^2}{R_0^2}) \frac{\mu}{\mu_0}} \quad (14)$$

An approximate value refers to ( $\mu = \mu_r \mu_0 \gg \mu_0$ ).

From eq. (14) the needed cylinder shield thickness can be determined ( $d$ ), depending on the given radius value  $R_i$  and equation (per unit) values of reduced field influence  $h_{i.pu.} = \frac{H_i}{H_0} 100\%$ . The

approximate value of shield thickness refers to ( $\mu \gg \mu_0$ ).

$$\frac{d}{R_i} = \frac{1}{\sqrt{1 - \frac{4}{\mu_r} \frac{100 - h_{i.pu.}}{h_{i.pu.}}}} \quad (15)$$



**Fig. 3. Needed cylinder shield thickness in relation to the expected loss of exterior magnetic field influence**

Fig. 3. presents geometric dimensions of a magnetic cylinder shield and position of fictive (centric) and three-phase (eccentric) conductors with a current  $i_A, i_B, i_C$  which at points  $M(r, (\rho), \theta)$  create adequate induction fields  $B_{n,p}$

Based on relationship (15), Fig. 3 shows dependability of the expected shield thickness ( $d$ ) with diameter ( $D_1$ ) and needed protection efficiency from the external field influence ( $\vec{H}_0$ ). As weakening of field by a few percent is noted, cylindric shield thickness of several millimeters is needed. As inside cylinder diameter is bigger the thicker shield is needed. For reducing unknown field influence on interior cylinder field to ( $\vec{H}_i = 5\% \vec{H}_0$ ), in a steel cylinder with a diameter of

( $D_1 = 1m$ ), shield thickness of ( $d = 4.5cm$ ) is needed. The chosen example demonstrates how difficult and costly is to attain protection using cylindric magnetic shields positioned horizontally in relation to the unknown field line ( $\vec{H}_0$ ).

*Eddy current effect:* if inside the very cylinder, eddy current is induced on its surface area, then ( $rot \vec{H} \neq 0$ ), that is ( $rot \vec{H} = \vec{J}$ ), and this value in a cylindrical coordinate system amounts to:

$$\begin{aligned} (rot \vec{H} = \vec{J}), \Leftrightarrow rot \vec{H} &= \frac{1}{r} \frac{\partial \varphi_m}{\partial \theta} \Big|_{r > R_o} - \frac{1}{r} \frac{\partial \varphi_m}{\partial \theta} \Big|_{r < R_i} = \vec{J} \\ \vec{J} = \sigma \cdot \vec{E} &= \frac{1}{r} \frac{\partial \varphi_m}{\partial \theta} \Big|_{r > R_o} - \frac{1}{r} \frac{\partial \varphi_m}{\partial \theta} \Big|_{r < R_i} \end{aligned} \quad (16)$$

Induced voltage in contour of a shell which passes through arbitrary point ( $M(r, \theta)$ ), Fig.2.b :

$$\oint_l \vec{E} \cdot d\vec{l} = -\frac{\partial \phi}{\partial t}, \quad rot \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \Leftrightarrow \phi = \int_S \vec{B} \cdot \vec{n} \cdot d\vec{s}; \quad (17)$$

Further, electric field strength ( $\vec{H} = H_m e^{j\omega t}$ ,  $\frac{\partial \vec{H}}{\partial t} = j\omega H_m e^{j\omega t}$ ), due to alternating current influence in a conductor on cylinder axis, instead of with a large letter H, is denoted with a small letter  $h = H_m \sin(2\pi f t)$ .

$\varphi_m = -h_i \cdot r \cdot \cos \theta$  - potential ( $\varphi_m$ ), and field strength ( $h_i$ ), inside shield screen,

$\varphi_m = -h_0 \cdot r \cdot \cos \theta + \frac{C_{lm} \cdot \cos \theta}{r}$  - potential  $\varphi_m$  and field strength ( $h_0$ ) outside a shield.

The magnet induction vector ( $\vec{B}$ ), can be expressed through magnetic field strength vector ( $\vec{H}$ ), and after replacing by (17) in a cylindrical coordinate system amounts to:

$$\oint_l \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \int_S \mu \cdot \vec{H} \cdot \vec{n} \cdot ds \Leftrightarrow \oint_l \vec{E} \cdot d\vec{l} = -\mu \frac{\partial}{\partial t} \int_S \int_{\theta} \frac{\partial h_i}{\partial t} \cdot \cos \theta \cdot r \cdot ds \cdot d\theta, \quad (18)$$

By differentiating a relationship (18)  $\frac{\partial}{\partial t} \left| \oint_l \vec{E} \cdot d\vec{l} \right| = \vec{E}$  we get.

$$\vec{E} = -\mu \cdot \frac{D}{2} \frac{\partial h_i}{\partial t} \cdot \sin \theta \quad (19)$$

Ohm's low in its elementary form is ( $\vec{J} = \sigma \cdot \vec{E} = \frac{1}{\rho} \vec{E}$ ), and if replaced by (19)

$$\vec{j} = \frac{\vec{J}}{d} = \frac{1}{\rho} \vec{E} = -\frac{\mu}{\rho} \cdot \frac{D}{2} \cdot \sin \theta \cdot \frac{\partial h_i}{\partial t} \quad (20)$$

So the equation for eddy current density on external surface area of a cylindrical shield is

$$(\vec{J} = \vec{j}_{pu} \cdot d), \vec{J} = \vec{j}_{pu} \cdot d = \frac{1}{\rho} \vec{E} = -\frac{\mu}{\rho} \cdot \frac{D \cdot d}{2} \cdot \sin \theta \cdot \frac{\partial h_i}{\partial t} \Leftrightarrow J = -\frac{\mu D \cdot d}{\rho \cdot 2} \sin \theta \cdot j\omega H_{im} \quad (20.a)$$

If we apply conditions for eq. (12) and eq. (13) in combination with eq. (20) and eq. (20.a.) the integration constant is determined  $C_{Int.}$

$$C_{Int.} = (h_i - h_0) \cdot R_0^2 \quad (21)$$

$$\frac{H_{im}}{H_{0m}} = \frac{1}{(1 + \sqrt{-1} \cdot \delta)} [\delta = (\pi \cdot f \cdot \mu \cdot dD / 2\rho)]; \sqrt{-1} = j \quad (22)$$

Here the maximum magnetic field strength values amount to:  $H_{im}$  - inside a shield and  $H_{0m}$  - outside a shield.

From bordering conditions eq. (16) and eq. (11), eq. (11.a) and eq. (22) the current density value is derived:

$$j = \frac{2H_0 \cdot \delta}{\sqrt{1 + \delta^2}} \sin \theta \quad (23)$$

By definition, the protection index by applying a shield with a magnetic parameter is a relation of induction  $B_i$  which exists on a shield and the magnetic induction value  $B_0$  which would act in a point if there were no shields:

$$k_a = \frac{B_i}{B_0} = \frac{1}{\sqrt{1 + \delta^2}} = \frac{1}{\sqrt{1 + (2\pi T_a)^2}} \quad (24)$$

$T_e = \frac{\mu D \cdot d}{\rho 4}$  - is a time constant which indicates eddy current weakening in a shield.

Now we can analyse an example where unknown field  $H_0$  in a space around a cylinder is created by a conductor placed on cylinder axis, Fig. 2, with a short circuit current  $i_{sh-cir.}$  Taking a restrained component into consideration in the first few periods, short circuit transient current can be demonstrated in a relationship [2] :

$$i_{sh-cir} = I_0 (e^{-\frac{t}{T_d}} - e^{-\frac{t}{T_a}} \cos \omega t) \quad (25)$$

- $i_{sh-cir.}$  - current value of a short circuit current,
- $T_d$  - time constant of direct current component,
- $T_a$  - time constant of a weakening component of alternating eddy current,
- $I_0$  - initial value of direct short circuit current,
- $t$  - time from the moment of emergence of short circuit current,
- $\omega = 2\pi f$  - cyclical frequency of network voltage,

Time changes of the magnetic induction component and the magnetic field strength component which emerged due to a change in direct current component, can be described in an equation:

$$b_0 = B_0 e^{-\frac{t}{T_d}} \Leftrightarrow h_0 = H_0 e^{-\frac{t}{T_d}} \quad (26)$$

Based on a relationship eq. (11), eq. (16), eq. (20), eq. (21) value derived is:

$$h_0 - h_i = \frac{\mu d \cdot D}{\rho 4} \frac{\partial h_i}{\partial t}, \quad H_0 e^{-\frac{t}{T_d}} - h_i = \frac{\mu d \cdot D}{\rho 4} \frac{\partial h_i}{\partial t} = T_e \frac{\partial h_i}{\partial t}$$

Differential equation solution:  $T_e \frac{\partial h_i}{\partial t} + h_i - H_0 e^{-\frac{t}{T_d}} = 0$

$$\text{are } h_i = H_0 \left[ \frac{e^{-\frac{t}{T_d}} - e^{-\frac{t}{T_e}}}{1 - \frac{T_e}{T_d}} \right], \text{ that is } b_i = B_0 \left[ \frac{e^{-\frac{t}{T_d}} - e^{-\frac{t}{T_e}}}{1 - \frac{T_e}{T_d}} \right] \quad (27)$$

The maximum value of a field component which emerged under the influence of direct current component is attained when a first extraction of function eq. (27) is equated to zero and occurring in a moment of:

$$t_m = \frac{T_d \cdot T_e}{T_d - T_e} \cdot \ln \frac{T_d}{T_e} \quad (28)$$

If the value eq. (28) is replaced by eq. (24) we get protection index of a field component which was created under direct current influence:

$$k_d = \frac{B_i}{B_0} = \left( \frac{T_e}{T_d} \right) \cdot e^{\left( \frac{T_e}{T_d - T_e} \right)} \quad (29)$$

The resultant protection effect of a ferromagnetic shield is determined by determining protective factors-for every current component individually after a time  $t_m$ . The maximum value of the electrodynamic force created in a moment when direct current component attains its maximum

value. Alternating current has a value eq. (30) and direct current eq. (31):

$$I_{a.pu} = \frac{i_a}{I_0} = e^{-\frac{tm}{Ta}} \quad (30)$$

$$I_{d.pu} = \frac{i_d}{I_0} = e^{-\frac{tm}{Td}} \quad (31)$$

$I_{a.pu}$  – unit value of short circuit alternating current component,  
 $I_{d.pu}$  – unit value of short circuit direct current component.

The alternating current induction is reduced to ( $b_a = k_a B_0$ ) and direct current to ( $b_d = k_d B_0$ ). The short circuit current has two components:  $i_{sh.cir} = i_d - i_a = I_0 (I_{d.pu} - I_{a.pu} \cos \omega t)$  (4').

The magnetic field emerged due to these currents is calculated based on equation:

$$b = B_0 (1 - \cos \omega t) \Leftrightarrow b = b_d - b_a = k_d B_0 - k_a B_0 \cos \omega t = B_0 (k_d - k_a \cos \omega t) \quad (5')$$

$$\text{Equation (6) becomes } f = \vec{F} / l \cong B_0 \cdot I_0 (I_d - I_a \cdot \cos \omega t) (k_d - k_a \cdot I_a \cdot \cos \omega t) \quad (6')$$

After replacing values from eq. (8) tj.  $B = \frac{I_0}{2\pi a}$  i uz  $I_0 = \sqrt{2} \cdot I_{sh-cir}$ , and trigonometric development of relationship (6'):

$$f = \frac{2 \cdot I_{sh-cir}^2}{a} \left[ \frac{2 \cdot (k_a I_{d.pu} + \frac{k_a \cdot I_{a.pu}^2}{2})}{3} - \frac{2}{3} (k_d + k_a I_{d.pu}) I_{a.pu} \cos \omega t + \frac{k_a I_{a.pu}^2}{3} \cos \omega t \right] \cdot 10^{-7} \quad (32)$$

$$f = \frac{2 I_{sh-cir}^2}{a} \left[ \frac{2 (k_a e^{-\frac{tm}{Td}} + \frac{k_a \cdot e^{-\frac{2tm}{Ta}}}{2})}{3} - \frac{2}{3} (k_d + k_a e^{-\frac{tm}{Td}}) e^{-\frac{tm}{Ta}} \cos \omega t + \frac{k_a e^{-\frac{2tm}{Ta}}}{3} \cos \omega t \right] \cdot 10^{-7} \quad (32')$$



## 2.1 The Force at the Three-pole (Three-phase) Conductors Neglecting the Eddy Current Influence

In a normal operating regime, Fig. 1b. when a three-phase conductor in a three-phase system encompasses one magnetic cylindrical shield resultant field of a system is very small. The magnetic flux in a shield closes, not allowing emergence of any significant eddy current values. In the case of extreme transient disturbances, three-phase short circuit, aperiodic components, that is, direct current components, receive big eddy current values in certain phases A, B, C, [2]:

$$i_{sh-cirA} = \sqrt{2} \cdot \ddot{I}_{sh-cir} \left( e^{-\frac{t}{Ta}} - \cos \omega t \right) \quad (33')$$

$$i_{sh-cirB} = \sqrt{2} \cdot \ddot{I}_{sh-cir} \left[ -\frac{1}{2} e^{-\frac{t}{Ta}} - \cos \left( \omega t - \frac{2\pi}{3} \right) \right] \quad (33'')$$

$$i_{sh-cirC} = \sqrt{2} \cdot \ddot{I}_{sh-cir} \left[ -\frac{1}{2} e^{-\frac{t}{Ta}} - \cos \left( \omega t + \frac{2\pi}{3} \right) \right] \quad (33''')$$

When these relationships develop, each one follows individually as,

$$i_{sh-cirA} = \sqrt{2} \cdot \ddot{I}_{sh-cir} \left( e^{-\frac{t}{Ta}} - \cos \omega t \right) \quad (34')$$

$$i_{sh-cirB} = \frac{\sqrt{2}}{2} \cdot \ddot{I}_{sh-cir} \left[ \cos \omega t - \sqrt{3} \sin \omega t - e^{-\frac{t}{Ta}} \right] \quad (34'')$$

$$i_{sh-cirC} = \frac{\sqrt{2}}{2} \cdot \ddot{I}_{sh-cir} \left[ \cos \omega t + \sqrt{3} \sin \omega t - e^{-\frac{t}{Ta}} \right] \quad (34''')$$

The total force per unit length on phase conductor A, created due to currents B,C, is:

$$f_{AX} = -\frac{\sqrt{3}}{2} \frac{\mu_0}{2\pi} i_{sh-cir.A} \left( \frac{i_{sh-cir.B}}{a_{AB}} + \frac{i_{sh-cir.C}}{a_{AC}} \right) \quad (35''')$$

$$f_{AY} = \frac{1}{2} \frac{\mu_0}{2\pi} i_{sh-cir.A} \left( \frac{i_{sh-cir.B}}{a_{AB}} + \frac{i_{sh-cir.C}}{a_{AC}} \right) \quad (35IV)$$

The total unit length affecting a phase A conductor is calculated according to eq. (35''') and eq. (35IV) is:

$$|f_A| = \sqrt{f_{AX}^2 + f_{AY}^2} = \frac{\mu_0}{2\pi} i_{sh-cir.A} \left( \frac{i_{sh-cir.B}}{a_{AB}} + \frac{i_{sh-cir.C}}{a_{AC}} \right) \quad (36)$$

## 2.2 Electrodynamic Forces of Three-pole Cylinder Shell Conductors with Eddy Current Influence

Short circuit currents flow through three eccentrically positioned conductors in Fig.3. and create magnetic induction, that is, fluxes in a cylindrical shield.

The resultant flux susceptible to time changes which indicates eddy current in a cylinder agrees with alternating currents. The needed values of induction and fluxes, of significant importance for eddy current and electrodynamic forces, can be determined using a method based on magnetic vector

potential calculations and Poisson differential equation,  $\Delta \vec{A} = -\mu \cdot \vec{J}$  [1,3]. This operation is very complex because it requires identification of a large number of bordering conditions and acceptance of value superposition of multi-phase conducting structures. The extent of eddy current influence in this type of three-phase eccentric structures can be determined through a simpler operation which is published in [6,8].

The geometric structures in Fig. 3. correspond to the following relationships:

$$\begin{aligned} \vec{B}_{n,\rho,A} &= \mu \cdot \vec{H}_{n,\rho,A} = \mu \frac{\vec{i}_A}{2\pi\rho_A}, & d\phi_{AM} &= \vec{B}_{n,\rho,A} \cdot d\vec{S} = \mu \frac{i_A}{2\pi\rho_A} l \cdot dr \cdot \cos\gamma, \\ \rho_A \cdot \cos\gamma + a \cdot \cos\theta_A &= r \Leftrightarrow \cos\gamma = \frac{r - a \cdot \cos\theta_A}{\rho_A}, \\ \rho_A^2 &= r^2 + a^2 - 2ra \cdot \cos\theta_A \end{aligned} \quad (37)$$

$$\begin{aligned} d\phi_{AM} &= \mu \frac{i_A}{2\pi\rho_A} \cos\gamma \cdot l \cdot dr = \mu \frac{i_A \cdot l}{2\pi\rho_A} \cdot \frac{r - a \cdot \cos\theta_A}{\rho_A} dr = \mu \frac{i_A \cdot l}{2\pi} \cdot \frac{r - a \cdot \cos\theta_A}{r^2 + a^2 - 2ra \cdot \cos\theta_A} dr, \\ \phi_{AM} &= \mu \frac{i_A \cdot l}{2\pi} \int_{R_i}^{R_o} \frac{r - a \cdot \cos\theta_A}{r^2 + a^2 - 2ra \cdot \cos\theta_A} dr = \mu \frac{i_A \cdot l}{4\pi} \left| \ln(r^2 + a^2 - 2ra \cdot \cos\theta_A) \right|_{R_i}^{R_o}, \\ \phi_{AM} &= \mu \frac{i_A \cdot l}{4\pi} \cdot \ln \frac{R_o^2 + a^2 - 2 \cdot R_o a \cdot \cos\theta_A}{R_i^2 + a^2 - 2 \cdot R_i a \cdot \cos\theta_A} \end{aligned} \quad (38)$$

The influence of currents in phases B and C can be determined in a similar way, that is, through their fluxes in a cylinder.

$$\begin{aligned} \phi_{BM} &= \mu \frac{i_B \cdot l}{4\pi} \cdot \ln \frac{R_o^2 + a^2 - 2 \cdot R_o a \cdot \cos\theta_B}{R_i^2 + a^2 - 2 \cdot R_i a \cdot \cos\theta_B} \\ \phi_{CM} &= \mu \frac{i_C \cdot l}{4\pi} \cdot \ln \frac{R_o^2 + a^2 - 2 \cdot R_o a \cdot \cos\theta_C}{R_i^2 + a^2 - 2 \cdot R_i a \cdot \cos\theta_C} \end{aligned}$$

The eddy current in a magnetic cylinder creates a resultant magnetic flux through elementary cross section to whom points  $M(r, \rho, \theta)$  belong to, according to relationship:

$$\phi_{\Sigma} = \phi_{AM} + \phi_{BM} + \phi_{CM} \quad (39)$$

Following this, adequate electrdynamic forces can be determined.

### 3. CONCLUSION

All conductors placed in a cylindrical metal magnetic shield must, due to developed electrodynamics force influence, be dimensioned in such a way as to be able to withstand short circuit currents in a short time span without

deforming or being permanently damaged When short circuits appear in shields which encompass three-phase conductors, very big electrodynamic force can be produced due to the short distance between the conductors.

The electrodynamic forces (eddy current emergence), according to relationship eq. (32), are significantly lower in shields which encompass only one phase of a conductor due to a protective effect of a shield. The eddy currents, as is demonstrated in this paper, significantly reduce magnetic field intensity produced by currents in conductors. Due to this effect, main electrodynamic forces in one-phase structures with shields, don't affect the conductor, but only affect the shield.

### COMPETING INTERESTS

Authors have declared that no competing interests exist.

### REFERENCES

1. Jovan Surutka. Elektromagnetic, Građevinska knjiga. University in Belgrad. 1966;231-232 and 394-395.

2. Gunter G. Seiph. Electrical Installations Handbooks. Publicis MCD Verlag, John Wiley and Sons. ISBN 0-471-49435-6. 2000;122-128.
3. Jiří Kulda, Magnetické pole v silnoproudé elektrotechnice. ACADEMIA, Praha. 1974; 44-46 and 78-80.
4. Rozenblat MA. Magnitnie elementi avtomatiki i vičislitelnoi tehniki, "Science". Moskva. 1978;31-41 and 78-85.
5. Bjelic S, Bogićević Z. computer simulation of theoretical model of electromagnetic transient processes in power transformers. International Journal of Information Technology and Computer Science, (IJITCS). 2013;6(1):1-12.
6. Slobodan Bjelić. Analysis of electrical faults in the current transformer zero sequence and similar devices. II. Jugoslovenski simposium of relay protection and automation, Ljubljana. 1980;155-161.
7. Bronštejn IN, Semendjajev KA. Applied mathematical engineering. Physical-Mathematical Literature, Moskva; 1962.
8. Slobodan Bjelić. The core form selection of a distant current converter in order to avoid magnetic influence. XXX Jugoslavian Conference ETAN, H. Novi. 1986;5:243-250.
9. Carter GW. Distribution of mechanical forces in magnetized material. IEEE. 1965; 9;112:33-41.
10. Gasparijan VR, Mnacakanjan MS. Elektromagnitnii sili v oblasti poluzakritoga paza elektriceskoi mashini. Električestvo. 1987;5:36-41.
11. Juan M. Romero, et al. Electrodynamics a la Horava. Modern Physics Letters A. 2010;25;3381. DOI: 10.1142/S0217732310034547.
12. Carpenter CJ, Ratti U. Calculation of mechanical forces in electrical machines by surface integration. L`Energia Electrica. 1974;5:51.
13. Kuznecov VA. Analiz formul dla opredelenija elektromagnitnih sil s pozicij teorii podobia. Električestvo. 1988;11.
14. Abramkin JV. Ob opredelenii elektromagnitnih sil v nelineinoj magnitnoi sisteme po ih objemnoi plotnostei. Električestvo. 1988;10.
15. Vas P. Artificial intelligence based electrical machines and drives. Oxford University Press; 1999.
16. Kulik A. Electrical machinery. Higher School, Russian. 1971;226.
17. Vučković V. The general theory of electrical machines. Science, Belgrade, Serbia; 1992.

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