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Exact Solutions and Rogue Waves for the Derivative Nonlinear Schrödinger Equation

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Abstract

Exact solutions which contain periodic solutions, soliton solutions and rogue wave solutions for two the modified derivative nonlinear Schrödinger equations, are obtained by means of solutions of a known derivative nonlinear Schrödinger equation. Two solutions' images are displayed, which can help one understand their dynamical behavior better. These results enrich the solutions' structural diversity for the modified derivative nonlinear schrödinger equations.

Keywords: The derivative nonlinear Schrödinger equation; The similarity transformation; Exact solution; Rogue wave solution.

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1 Introduction

The derivative nonlinear Schrödinger equation(DNLSE)

$$iu_t + \varepsilon u_{xx} + i\alpha(|u|^2u)_x = 0, \tag{1}$$

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where $\varepsilon = \pm 1$, $\alpha \neq 0$ is a arbitrary constant. This equation has a rich application background in plasma physics and nonlinear optics [1-4], and has been studied by many scholars from two aspects: mathematical theory and physical application [1-10].

When $\varepsilon = -1$ and $\alpha = 1$, Eq.(1) is rewritten into

$$iQ_T - Q_{XX} + i(|Q|^2 Q)_X = 0, \tag{2}$$

Steudel H obtained the following exact solutions for Eq.(2)[9]

$$Q(X,T) = -\frac{4MN(M\cos(4NM(X-4(N^2+M^2)T))+iN\sin(4NM(X-4(N^2+M^2)T)))^3}{(M^2+(N^2-M^2)\sin^2(4NM(X-4(N^2+M^2)T)))^2} e^{(-i(2(N^2+M^2)X-4(N^4+M^4+6N^2M^2)T))}, \tag{3}$$

and

$$Q(X,T) = \frac{4MN(M\cosh(4NM(X-4(N^2-M^2)T))+iN\sinh(4NM(X-4(N^2-M^2)T)))^3}{(M^2+(N^2+M^2)\sinh^2(4NM(X-4(N^2-M^2)T)))^2} e^{(-i(2(N^2-M^2)X-4(N^4+M^4-6N^2M^2)T))}, \tag{4}$$

where M, N are arbitrary constants.

Xu S. W. *et al* constructed following rogue wave solutions for Eq.(2)[10]

$$Q(X,T) = \frac{4K(4iK^2(4K^2T-X)-1)^3}{(16K^4(4K^2T-X)^2+1)^2} e^{-2iK^2(X-2K^2T)} \tag{5}$$

where K is a arbitrary constant.

Many researchers have studied the modified derivative nonlinear Schrödinger equation(MDNLSE)

$$iu_t = u_{xx} + i\alpha(|u|^2 u)_x + 2(|u|^2 - \omega)u, \tag{6}$$

where α and ω are real constants, which has applications in nonlinear optics [11,12] and plasma physics [13]. Rich results have been obtained for MDNLSE, such as the homoclinic orbit, etc.[14-17]. However, as far as we know, the rogue wave solution of Eq.(6) has not been reported yet.

In present work, we consider the following modified derivative nonlinear Schrödinger equation

$$iu_t + \varepsilon u_{xx} + i\delta\alpha(|u|^2 u)_x + 2\beta|u|^2 u + 2\gamma\omega u = 0, \tag{7}$$

where $\varepsilon = \pm 1, \delta = \pm 1$, α, β, γ and ω are real constants. Obviously, Eq.(7) contains Eq.(1) and Eq.(6). We will convert Eq.(7) into Eq.(2), and then obtain the solutions of Eq.(7) through expressions (3) to (5). So, periodic solutions, soliton solutions and rogue wave solutions for Eq.(1) and Eq.(6) will be obtained, which digital images will also be displayed.

2 The Similarity Transformation and Exact Solutions for Eq.(7)

First, we use a similarity transformation to convert Eq.(7) into Eq.(2)[18]. Assume

$$u(x,t) = PQ(X(x,t), T(t))e^{i\varphi(x,t)}, \tag{8}$$

where P is a constant to be determined, $X(x,t), T(t)$ and $\varphi(x,t)$ are the functions to be determined. Substituting Eq.(8) into Eq.(7) (the tedious calculation process is omitted), we can obtain solutions about $\varphi(x,t), X(x,t)$ and $T(t)$ as follows

$$\varphi(x,t) = \frac{2\delta\alpha\beta x + 2(\delta^2\alpha^2\omega\gamma - 2\varepsilon\beta^2)t}{\delta^2\alpha^2} + C_1,\ X(x,t) = -\frac{\delta\alpha P^2}{\varepsilon}x + 4\beta P^2 t + C_2,\ T(t) = -\frac{\delta^2\alpha^2 P^4}{\varepsilon}t + C_2, \tag{9}$$

where P, C_1 and C_2 are arbitrary real constants.

Now, we substitute Eq.(9) into Eq.(3)-Eq.(5), periodic solution, soliton solution and rogue wave solution are obtained for Eq.(7), which are written as follows

Periodic solution:

$$u(x,t) = -\frac{4MN(M\cos(K(x,t))+iN\sin(K(x,t)))^3}{(M^2+(N^2-M^2)\sin^2(K(x,t)))^2}e^{-i\psi(x,t)}. \tag{10}$$

where $K(x,t) = 4NM(-\frac{\delta\alpha P^2}{\varepsilon}x + 4\beta P^2 t + C_2 - 4(N^2+M^2)(-\frac{\delta^2\alpha^2P^4}{\varepsilon}t + C_2))$, $\psi(x,t) = 2(N^2 + M^2)(-\frac{\delta\alpha P^2}{\varepsilon}x + 4\beta P^2 t + C_2) - 4(N^4+M^4+6N^2M^2)(-\frac{\delta^2\alpha^2P^4}{\varepsilon}t + C_2) + \frac{2\delta\alpha\beta x + 2(\delta^2\alpha^2\omega\gamma - 2\varepsilon\beta^2)t}{\delta^2\alpha^2} + C_1$.

soliton solution:

$$u(x,t) = \frac{4MN(M\cosh(L(x,t))+iN\sinh(L(x,t)))^3}{(M^2+(N^2+M^2)\sinh^2(L(x,t)))^2}e^{-i\phi(x,t)}, \tag{11}$$

where $L(x,t) = 4NM(-\frac{\delta\alpha P^2}{\varepsilon}x + 4\beta P^2 t + C_2 - 4(N^2-M^2)(-\frac{\delta^2\alpha^2P^4}{\varepsilon}t + C_2))$, $\phi(x,t) = 2(N^2 - M^2)(-\frac{\delta\alpha P^2}{\varepsilon}x + 4\beta P^2 t + C_2) - 4(N^4+M^4-6N^2M^2)(-\frac{\delta^2\alpha^2P^4}{\varepsilon}t + C_2) + \frac{2\delta\alpha\beta x + 2(\delta^2\alpha^2\omega\gamma - 2\varepsilon\beta^2)t}{\delta^2\alpha^2} + C_1$.

Rogue wave solution:

$$\begin{aligned} u(x,t) = &\frac{4K(4iK^2(4K^2(-\frac{\delta^2\alpha^2P^4}{\varepsilon}t+C_2)-(-\frac{\delta\alpha P^2}{\varepsilon}x+4\beta P^2t+C_2))-1)^3}{(16K^4(4K^2(-\frac{\delta^2\alpha^2P^4}{\varepsilon}t+C_2)-(-\frac{\delta\alpha P^2}{\varepsilon}x+4\beta P^2t+C_2))^2+1)^2} \\ &e^{-i(2K^2((-\frac{\delta\alpha P^2}{\varepsilon}x+4\beta P^2t+C_2)-2K^2(-\frac{\delta^2\alpha^2P^4}{\varepsilon}t+C_2))+\frac{2\delta\alpha\beta x+2(\delta^2\alpha^2\omega\gamma-2\varepsilon\beta^2)t}{\delta^2\alpha^2}+C_1)}. \end{aligned} \tag{12}$$

In Eq.(10) - Eq.(12), M, N, K, P, C_1 and C_2 are arbitrary real constants.

3 Solutions for Eq.(1) and Eq.(6)

In Eq.(10) - Eq.(12), setting $\delta = 1$ and $\beta = \gamma = 0$, three solutions of Eq.(1) are obtained as follows

$$u_1(x,t) = -\frac{4MN(M\cos(K(x,t))+iN\sin(K(x,t)))^3}{(M^2+(N^2-M^2)\sin^2(K(x,t)))^2}e^{-i\psi(x,t)}, \tag{13}$$

where $K(x,t) = 4NM(-\frac{\alpha P^2}{\varepsilon}x + C_2 - 4(N^2+M^2)(-\frac{\alpha^2P^4}{\varepsilon}t + C_2))$ and $\psi(x,t) = 2(N^2+M^2)(-\frac{\alpha P^2}{\varepsilon}x + C_2) - 4(N^4+M^4+6N^2M^2)(-\frac{\alpha^2P^4}{\varepsilon}t + C_2) + C_1$.

$$u_2(x,t) = \frac{4MN(M\cosh(L(x,t))+iN\sinh(L(x,t)))^3}{(M^2+(N^2+M^2)\sinh^2(L(x,t)))^2}e^{-i\phi(x,t)}, \tag{14}$$

where $L(x,t) = 4NM(-\frac{\alpha P^2}{\varepsilon}x + C_2 - 4(N^2-M^2)(-\frac{\alpha^2P^4}{\varepsilon}t + C_2))$, $\phi(x,t) = 2(N^2-M^2)(-\frac{\alpha P^2}{\varepsilon}x + C_2) - 4(N^4+M^4-6N^2M^2)(-\frac{\alpha^2P^4}{\varepsilon}t + C_2) + C_1$.

$$\begin{aligned} u_3(x,t) = &\frac{4K(4iK^2(4K^2(-\frac{\delta^2\alpha^2P^4}{\varepsilon}t+C_2)-(-\frac{\delta\alpha P^2}{\varepsilon}x+4\beta P^2t+C_2))-1)^3}{(16K^4(4K^2(-\frac{\delta^2\alpha^2P^4}{\varepsilon}t+C_2)-(-\frac{\delta\alpha P^2}{\varepsilon}x+4\beta P^2t+C_2))^2+1)^2}\times \\ &e^{-i(2K^2((-\frac{\delta\alpha P^2}{\varepsilon}x+4\beta P^2t+C_2)-2K^2(-\frac{\delta^2\alpha^2P^4}{\varepsilon}t+C_2))+\frac{2\delta\alpha\beta x+2(\delta^2\alpha^2\omega\gamma-2\varepsilon\beta^2)t}{\delta^2\alpha^2}+C_1)}. \end{aligned} \tag{15}$$

Similarly, in Eq.(10) - Eq.(12), setting $\varepsilon = -1, \delta = -1, \beta = -1$ and $\gamma = 1$, three solutions of Eq.(6) are obtained as follows

$$u_1(x,t) = -\frac{4MN(M\cos(K(x,t))+iN\sin(K(x,t)))^3}{(M^2+(N^2-M^2)\sin^2(K(x,t)))^2}e^{-i\psi(x,t)}. \tag{16}$$

where $K(x,t) = 4NM(-\alpha P^2x - 4P^2t + C_2 - 4(N^2 + M^2)(\alpha^2P^4t + C_2))$, $\psi(x,t) = 2(N^2 + M^2)(-\alpha P^2x - 4P^2t + C_2) - 4(N^4 + M^4 + 6N^2M^2)(\alpha^2P^4t + C_2) + \frac{2\alpha x + 2(\alpha^2\omega + 2)t}{\alpha^2} + C_1$.

$$u_2(x,t) = \frac{4MN(M\cosh(L(x,t)) + i\,N\sinh(L(x,t)))^3}{(M^2 + (N^2+M^2)\sinh^2(L(x,t)))^2}e^{-i\phi(x,t)}, \tag{17}$$

where $L(x,t) = 4NM(-\alpha P^2x - 4P^2t + C_2 - 4(N^2 - M^2)(\alpha^2P^4t + C_2))$, $\phi(x,t) = 2(N^2 - M^2)(-\alpha P^2x - 4P^2t + C_2) - 4(N^4 + M^4 - 6N^2M^2)(\alpha^2P^4t + C_2) + \frac{2\alpha x + 2(\alpha^2\omega + 2)t}{\alpha^2} + C_1$.

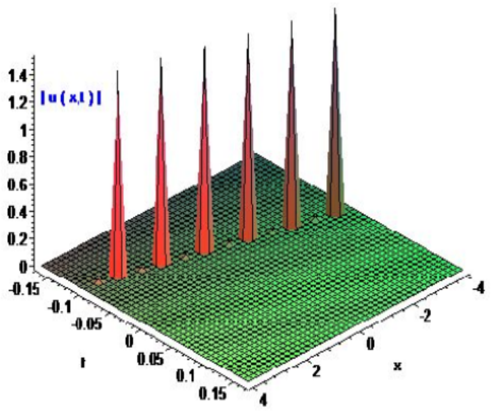

Fig.1. Behaviour of $|u(x,t)|$ in Eq.(17) with $M = 2, N = 3, P = 2, \alpha = 1, \omega = 2, C_1 = 2$ and $C_2 = 1$.

$$u_3(x,t) = \frac{4K(4iK^2(4K^2(\alpha^2P^4t+C_2) - (-\alpha P^2x - 4P^2t + C_2)) - 1)^3}{(16K^4(4K^2(\alpha^2P^4t+C_2) - (-\alpha P^2x - 4P^2t + C_2))^2 + 1)^2} \times e^{-i(2K^2((-\alpha P^2x - 4P^2t + C_2) - 2K^2(\alpha^2P^4t + C_2)) + \frac{2\alpha x + 2(\alpha^2\omega+2)t}{\alpha^2} + C_1)}. \tag{18}$$

From Fig.1, we clearly see that Eq.(17) ($u_2(x,t)$) is a soliton solution of Eq.(6), which maintains a steady state of motion.

Similarly, from Fig.2, we obviously see that Eq.(18) ($u_3(x,t)$) is a rogue wave structure, which produces large amplitude waves.

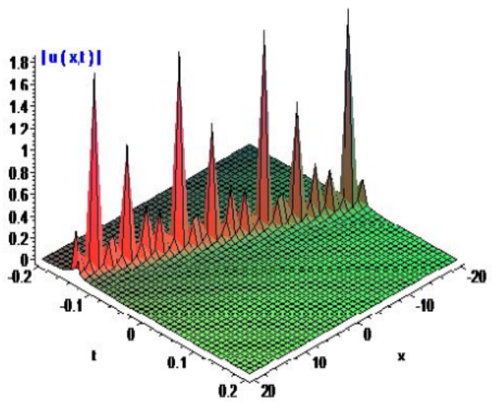

Fig.2. Behaviour of $|u(x,t)|$ in Eq.(18) with $K = 4, P = 2, \alpha = 1, \omega = 2, C_1 = 2$ and $C_2 = 1$.

4 Conclusion

Two the modified derivative nonlinear Schrödinger equations are unified into a equation to be solved, then their solutions are obtained by means of solutions of a known equation. The results show that two the modified derivative nonlinear Schrödinger equations have periodic solution, soliton solution and rogue wave solution. These solutions help us to understand the dynamics of these equations, and the problem-solving method can also guide the study of other equations.

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Competing Interests

Authors have declared that no competing interests exist.

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