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Riccati Differential Equations: A Computational Approach

J. Sunday[1*]

[1]Department of Mathematics, Adamawa State University, Mubi, Nigeria.

Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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ABSTRACT

One of the most important classes of nonlinear differential equations that have a great deal of applications is the Riccati Differential Equations (RDEs). In this paper, a quarter-step method is derived for the solution of RDEs by collocating and interpolating the Laguerre polynomial basis function. To establish the reliability and applicability of the method on RDEs, some model problems have been solved. The results obtained in terms of the point wise absolute errors show that the method developed approximates the exact solution closely. The research further investigated the basic properties of the method developed and found it to be zero-stable, consistent and convergent.

Keywords: Computational; nonlinear; quarter-step; RDEs.

2010 AMS subject classification: 65L05, 65L06, 65D30.

1. INTRODUCTION

The RDE named after the Italian nobleman Count Jacopo Francesco Riccati (1676-1754) find applications in random processes, optimal control and diffusion problem, [1]. Besides its applications in engineering and science that today are considered classical, the RDE is also applied in financial mathematics [2], robust stabilization, stochastic realization theory, network synthesis and optimal control [3]. Also, according to [4], the RDE is an essential tool for

**Corresponding author: Email: joshuasunday2000@yahoo.com;*

modeling many physical situations such as spring mass systems, resistor-capacitor-induction circuits, bending of beams, chemical reactions, pendulum, and the motion of rotating mass around body. The RDE are also found to be applicable in oscillations, [5].

In view of these applications, we are motivated to derive a computational method for the solution of RDEs of the form;

$$y'(t) = a(t) + b(t)y(t) + c(t)y^2(t),\ 0 \le t \le T \quad (1)$$

with initial conditions,

$$y(t_0) = y_0 \quad (2)$$

where $a(t), b(t), c(t)$ are continuous with $c(t) \neq 0$ and $t_0,\ y_0$ are arbitrary constants for $y(t)$ which is an unknown function.

The RDE in (1) can also be denoted by the equation below;

$$y'(t) = f(t, y) \quad (3)$$

The general solution of a class of RDE shall be presented below in the form of a theorem.

Theorem 1 [6]

Consider the RDE

$$y'(t) + p(t)y(t) - y^2(t) = q(t) \quad (4)$$

with the initial condition $y(t_0) = y_0$ for some initial value t_0. Assume that $q(t_0) = q_0 > 0$ and that the integral $\int_{t_0}^{t} p(\tau)d\tau$ exists. Assume further that the function $q(t)$ satisfies the relation

$$q(t) = \frac{q_0 e^{-2\int_{t_0}^{t} p(\tau)d\tau}}{\left(1 + K\sqrt{q_0}\int_{t_0}^{t} e^{\int_{t_0}^{t} -p(\tau)d\tau} d\lambda\right)^2} \quad (5)$$

for some constant K. Then, the general solution of (5) is given by,

$$y(t) = f(t)\sqrt{q(t)} \quad (6)$$

where the function $f(t)$ is given by,

$$f(t) = \begin{cases} -\dfrac{K}{2} + \dfrac{\alpha\left(1 + e^{\alpha\theta_1(t)}\right)}{2\left(1 - e^{\alpha\theta_1(t)}\right)}, & if \quad K^2 > 4 \\ -\dfrac{K}{2} + \dfrac{\beta}{2}\tan\left(\dfrac{\beta\theta_2(t)}{2}\right), & if \quad K^2 < 4 \\ -\dfrac{K}{2} - \dfrac{1}{\theta_3(t)}, & if \quad K^2 = 4 \end{cases}$$

and the functions $\theta_n(t);\ n = 1, 2, 3$ are given by

$$\theta_n(t) = c_n + \int_{t_0}^{t} \sqrt{q(t)}d\tau$$

where

$$c_1 = \frac{1}{\alpha}\ln\left(\frac{2y_0 + \sqrt{q_0}(K - \alpha)}{2y_0 + \sqrt{q_0}(K + \alpha)}\right)$$

$$c_2 = \frac{2}{\beta}\tan^{-1}\left(\frac{2y_0 + K\sqrt{q_0}}{\beta\sqrt{q_0}}\right)$$

$$c_3 = -\frac{2\sqrt{q_0}}{2y_0 + K\sqrt{q_0}}$$

and

$$\alpha = \sqrt{K^2 - 4}$$

$$\beta = \sqrt{4 - K^2}$$

See [6] for proof

The RDE has been studied by some researchers. They adopted different methods in solving the RDEs. These methods include the Adomian Decomposition Method (ADM) [7,8,9,10], Variational Iteration Method (VIM) [11,12,13,14,15,16,17], Chebyshev wavelets

[18], classical fourth order Runge-Kutta method [19], hybrid function and Tau method [20], Differential Transformation Method (DTM) [21,22], Non-Standard Finite Difference Method (NSFDM) [3], Homotopy Analysis Method (HAM) [23,24], Homotopy Perturbation Method (HPM) [8,25,26], among others.

It is important to state that the above mentioned methods have some setbacks in their performance on the RDEs. For instance, in applying the ADM, very complicated and tough Adomian polynomials have to be constructed which make the work cumbersome. In the VIM, identification of Lagrange multipliers yields an underlying accuracy. The HPM needs a linear functional equation in each iteration to solve nonlinear equations, forming these functional equations could be very difficult. The performance of HAM is very much dependent on the choice of the auxiliary parameter h of the zero-order deformation equation. Moreover, the convergence region and implementation of these results are very small.

In view of the foregoing, an alternative computational method shall be constructed in this research for the solution of RDEs of the form (1).

2. FORMULATION OF THE METHOD

A computational method of the form,

$$A^{(0)}\mathbf{Y}_m = E\mathbf{y}_n + hd\mathbf{f}(\mathbf{y}_n) + hb\mathbf{F}(\mathbf{Y}_m) \qquad (7)$$

will be developed for the solution of RDEs of the form (1), where $A^{(0)},\ E,\ d\ and\ b$ are $r \times r$ matrices (r is the number of collocation points). $Y_m,\ y_n,\ F(Y_m)\ and\ f(y_n)$ are vector matrices with r entries.

In doing this, the Laguerre polynomial shall be adopted as a basis function. The Laguerre polynomial is generally given by,

$$y(t) = \sum_{n=0}^{r+s-1} \frac{e^t}{n!}\frac{d^n}{dt^n}\left(t^n e^{-t}\right) \qquad (8)$$

where $r\ and\ s$ are the numbers of collocation and interpolation points respectively.

Let the approximate solution to (1) be given by Laguerre polynomial of degree 5, by allowing $r+s-1=5$ in equation (8), that is,

$$y(t) = \sum_{n=0}^{5}\left[\frac{e^t}{n!}\frac{d^n}{dt^n}\left(t^n e^{-t}\right)\right] = 720 - 1800t + 1200t^2 - 300t^3 + 30t^4 - t^5 \qquad (9)$$

with the first derivative given by,

$$y'(t) = -1800 + 2400t - 900t^2 + 120t^3 - 5t^4 \qquad (10)$$

Substituting (10) into (3) gives,

$$f(t,y) = -1800 + 2400t - 900t^2 + 120t^3 - 5t^4 \qquad (11)$$

Now, interpolating (9) at point $t_{n+s}, s=0$ and collocating (11) at points $t_{n+r}, r = 0\left(\frac{1}{16}\right)\frac{1}{4}$, leads to a system of nonlinear equation of the form,

$$TA = U \qquad (12)$$

where

$$A = \begin{bmatrix} a_0 & a_1 & a_2 & a_3 & a_4 & a_5 \end{bmatrix}^T \qquad U = \begin{bmatrix} y_n & f_n & f_{n+\frac{1}{16}} & f_{n+\frac{1}{8}} & f_{n+\frac{3}{16}} & f_{n+\frac{1}{4}} \end{bmatrix}^T$$

$$T=\begin{bmatrix} 720 & -1800t_n & 1200t_n^2 & -300t_n^3 & 30t_n^4 & -t_n^5 \\ 0 & -1800 & 2400t_n & -900t_n^2 & 120t_n^3 & -5t_n^4 \\ 0 & -1800 & 2400t_{n+\frac{1}{16}} & -900t^2_{n+\frac{1}{16}} & 120t^3_{n+\frac{1}{16}} & -5t^4_{n+\frac{1}{16}} \\ 0 & -1800 & 2400t_{n+\frac{1}{8}} & -900t^2_{n+\frac{1}{8}} & 120t^3_{n+\frac{1}{8}} & -5t^4_{n+\frac{1}{8}} \\ 0 & -1800 & 2400t_{n+\frac{3}{16}} & -900t^2_{n+\frac{3}{16}} & 120t^3_{n+\frac{3}{16}} & -5t^4_{n+\frac{3}{16}} \\ 0 & -1800 & 2400t_{n+\frac{1}{4}} & -900t^2_{n+\frac{1}{4}} & 120t^3_{n+\frac{1}{4}} & -5t^4_{n+\frac{1}{4}} \end{bmatrix}$$

Solving (12) by Gauss elimination method for the $a_j's, j=0(1)5$ and substituting back into the Laguerre polynomial basis function gives a linear multistep method of the form,

$$y(t)=\alpha_0(t)y_n+h\sum_{j=0}^{\frac{1}{4}}\beta_j(t)f_{n+j},\ j=0\left(\frac{1}{16}\right)\frac{1}{4} \tag{13}$$

where the coefficients of $y_n\ and\ f_{n+j}$ are given as,

$$\left.\begin{aligned} &\alpha_0=1\\ &\beta_0=\frac{1}{45}(24576x^5-19200x^4+5600x^3-750x^2+45x)\\ &\beta_{\frac{1}{16}}=\frac{32}{45}(-3072x^5+2160x^4-520x^3+45x^2)\\ &\beta_{\frac{1}{8}}=-\frac{8}{15}(-6144x^5+3840x^4-760x^3+45x^2)\\ &\beta_{\frac{3}{16}}=\frac{32}{45}(-3072x^5+1680x^4-280x^3+15x^2)\\ &\beta_{\frac{1}{4}}=-\frac{2}{45}(-12288x^5+5760x^4-880x^3+45x^2) \end{aligned}\right\} \tag{14}$$

and x is given by

$$x=\frac{t-t_n}{h} \tag{15}$$

Evaluating (13) at $t=\frac{1}{16}\left(\frac{1}{16}\right)\frac{1}{4}$, gives a discrete computational method of the form (7) given by,

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_{n+\frac{1}{16}} \\ y_{n+\frac{1}{8}} \\ y_{n+\frac{3}{16}} \\ y_{n+\frac{1}{4}} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_{n-\frac{3}{16}} \\ y_{n-\frac{1}{8}} \\ y_{n-\frac{1}{16}} \\ y_n \end{bmatrix} + h \begin{bmatrix} 0 & 0 & 0 & \frac{251}{11520} \\ 0 & 0 & 0 & \frac{29}{1440} \\ 0 & 0 & 0 & \frac{27}{1280} \\ 0 & 0 & 0 & \frac{7}{360} \end{bmatrix} \begin{bmatrix} f_{n-\frac{3}{16}} \\ f_{n-\frac{1}{8}} \\ f_{n-\frac{1}{16}} \\ f_n \end{bmatrix}$$

$$+ h \begin{bmatrix} \frac{323}{5760} & \frac{-11}{480} & \frac{53}{5760} & \frac{-19}{11520} \\ \frac{31}{360} & \frac{1}{60} & \frac{1}{360} & \frac{-1}{1440} \\ \frac{51}{640} & \frac{9}{160} & \frac{21}{640} & \frac{-3}{1280} \\ \frac{4}{45} & \frac{1}{30} & \frac{4}{45} & \frac{7}{360} \end{bmatrix} \begin{bmatrix} f_{n+\frac{1}{16}} \\ f_{n+\frac{1}{8}} \\ f_{n+\frac{3}{16}} \\ f_{n+\frac{1}{4}} \end{bmatrix} \tag{16}$$

3. ANALYSIS OF THE METHOD

Some basic properties of the computational method derived shall be discussed in this section.

3.1 Order of Accuracy of the Method

The linear operator of the computational method derived in equation (16) is expressed as,

$$L\{y(x);h\} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_{n+\frac{1}{16}} \\ y_{n+\frac{1}{8}} \\ y_{n+\frac{3}{16}} \\ y_{n+\frac{1}{4}} \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_{n-\frac{3}{16}} \\ y_{n-\frac{1}{8}} \\ y_{n-\frac{1}{16}} \\ y_n \end{bmatrix}$$

$$-h \begin{bmatrix} \frac{251}{11520} & \frac{323}{5760} & \frac{-11}{480} & \frac{53}{5760} & \frac{-19}{11520} \\ \frac{29}{1440} & \frac{31}{360} & \frac{1}{60} & \frac{1}{360} & \frac{-1}{1440} \\ \frac{27}{1280} & \frac{51}{640} & \frac{9}{160} & \frac{21}{640} & \frac{-3}{1280} \\ \frac{7}{360} & \frac{4}{45} & \frac{1}{30} & \frac{4}{45} & \frac{7}{360} \end{bmatrix} \begin{bmatrix} f_n \\ f_{n+\frac{1}{16}} \\ f_{n+\frac{1}{8}} \\ f_{n+\frac{3}{16}} \\ f_{n+\frac{1}{4}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{17}$$

Expanding (17) in Taylor series about x_n, we have

$$\begin{bmatrix} \sum_{j=0}^{\infty}\frac{\left(\frac{1}{16}h\right)^j}{j!}y_n^j - y_n - \frac{251}{11520}hy_n' - \sum_{j=0}^{\infty}\frac{h^{j+1}}{j!}\left\{\frac{323}{5760}\left(\frac{1}{16}\right)^j - \frac{11}{480}\left(\frac{1}{8}\right)^j + \frac{53}{5760}\left(\frac{3}{16}\right)^j - \frac{19}{11520}\left(\frac{1}{4}\right)^j\right\} \\ \sum_{j=0}^{\infty}\frac{\left(\frac{1}{8}h\right)^j}{j!}y_n^j - y_n - \frac{29}{1440}hy_n' - \sum_{j=0}^{\infty}\frac{h^{j+1}}{j!}\left\{\frac{31}{360}\left(\frac{1}{16}\right)^j + \frac{1}{60}\left(\frac{1}{8}\right)^j + \frac{1}{360}\left(\frac{3}{16}\right)^j - \frac{1}{1440}\left(\frac{1}{4}\right)^j\right\} \\ \sum_{j=0}^{\infty}\frac{\left(\frac{3}{16}h\right)^j}{j!}y_n^j - y_n - \frac{27}{1280}hy_n' - \sum_{j=0}^{\infty}\frac{h^{j+1}}{j!}\left\{\frac{51}{640}\left(\frac{1}{16}\right)^j + \frac{9}{160}\left(\frac{1}{8}\right)^j + \frac{21}{640}\left(\frac{3}{16}\right)^j - \frac{3}{1280}\left(\frac{1}{4}\right)^j\right\} \\ \sum_{j=0}^{\infty}\frac{\left(\frac{1}{4}h\right)^j}{j!}y_n^j - y_n - \frac{7}{360}hy_n' - \sum_{j=0}^{\infty}\frac{h^{j+1}}{j!}\left\{\frac{4}{45}\left(\frac{1}{16}\right)^j + \frac{1}{30}\left(\frac{1}{8}\right)^j + \frac{4}{45}\left(\frac{3}{16}\right)^j + \frac{7}{360}\left(\frac{1}{4}\right)^j\right\} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{18}$$

Hence,

$$\bar{c}_0 = \bar{c}_1 = \bar{c}_2 = \bar{c}_3 = \bar{c}_4 = \bar{c}_5 = 0, \quad \bar{c}_6 = \begin{bmatrix} 1.1176\times10^{-9} & 6.6227\times10^{-10} & 1.1176\times10^{-9} & -3.1537\times10^{-11} \end{bmatrix}^T$$

Therefore, the computational method (16) is of uniform order $p=5$ and the error constant is $[1.1176\times10^{-9} \;\; 6.6227\times10^{-10} \;\; 1.1176\times10^{-9} \;\; -3.1537\times10^{-11}]^T$

3.2 Consistency of the Method

The computational method (16) is consistent since it has uniform order $p = 5 \geq 1$.

3.3 Zero Stability of the Method

Definition 3.1 [27]: A block method is said to be zero-stable, if the roots $z_s, s = 1,2,...,k$ of the first characteristic polynomial $\rho(z)$ defined by $\rho(z) = \det(zA^{(0)} - E)$ satisfies $|z_s| \leq 1$ and every root satisfying $|z_s| = 1$ have multiplicity not exceeding the order of the differential equation.

For the computational method (16), the first characteristic polynomial is given by,

$$\rho(z) = \left| z\begin{bmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \end{bmatrix} - \begin{bmatrix} 0&0&0&1 \\ 0&0&0&1 \\ 0&0&0&1 \\ 0&0&0&1 \end{bmatrix} \right|$$

$$= \begin{vmatrix} z&0&0&-1 \\ 0&z&0&-1 \\ 0&0&z&-1 \\ 0&0&0&z-1 \end{vmatrix} = z^3(z-1)$$

Thus, solving for z in

$$z^3(z-1) = 0 \tag{19}$$

gives $z_1 = z_2 = z_3 = 0$ *and* $z_4 = 1$. Hence, the computational method (16) is zero-stable.

3.4 Convergence of the Method

The method (16) is convergent since it is consistent and zero-stable.

Theorem 3.1 [5]

A method is convergent if and only if it is zero stable and consistent.

3.5 Region of Absolute Stability of the Method

Applying the boundary locus method, the stability polynomial of the computational method (16) is given by,

$$\bar{h}(w) = -h^4\left(\frac{1}{327680}w^3 + \frac{1}{327680}w^4\right) - h^3\left(\frac{5}{24576}w^4 + \frac{5}{24576}w^3\right) - h^2\left(\frac{7}{1024}w^3 - \frac{7}{1024}w^4\right) - h\left(\frac{1}{8}w^4 + \frac{1}{8}w^3\right) + w^4 - w^3 \quad (20)$$

The region of absolute stability of the method is therefore shown in Fig. 1.

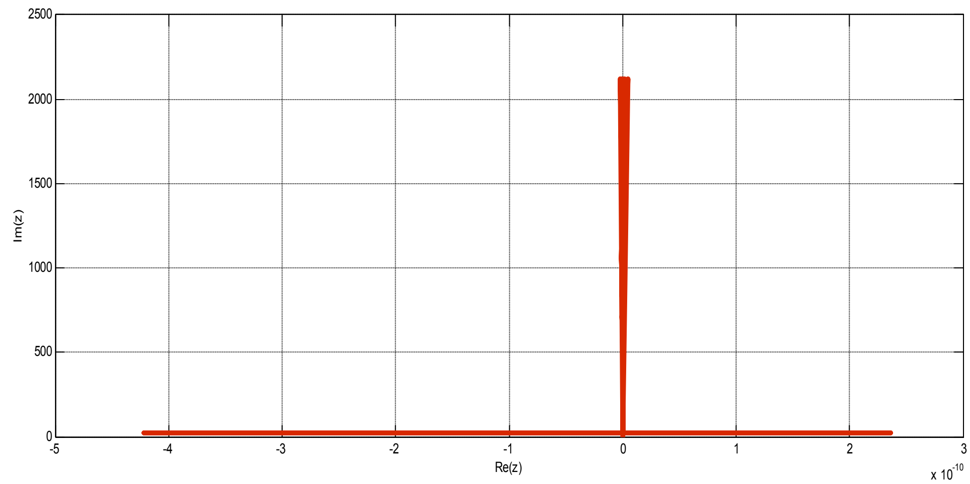

Fig. 1. Stability Region for the computational method

The RAS obtained in Fig. 1 is L-stable since it is A-stable and also encroaches into the positive half of the complex plane, [28].

4. RESULTS

4.1 Numerical Experiments

The computational method derived shall be applied on some modeled RDEs to test how reliable and efficient the method is.

The following notations shall be used in the Tables below:

ERR= Absolute error in the computational method
Eval *t* =Evaluation time per seconds
EFA-Absolute error in [19]
EYH-Absolute error in [20]
ENB-Absolute error in [25]

Problem 4.1:

Consider the Riccati differential equation,

$$y'(t) = 1 + 2y(t) - y^2(t) \quad (21)$$

with the initial conditions,

$$y(0) = 0 \quad (22)$$

The exact solution is given by,

$$y(t) = 1 + \sqrt{2}\tanh\left(\sqrt{2}t + \frac{1}{2}\log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)\right) \quad (23)$$

Source: [19]

Problem 4.2:

Consider the Riccati differential equation,

$$y'(t) = 1 - y^2(t) \quad (24)$$

with initial conditions,

$$y(0) = 0 \quad (25)$$

The exact solution to the problem is

$$y(t) = \frac{e^{2t}-1}{e^{2t}+1} \quad (26)$$

Source: [20]

Table 4.1. Showing the result for problem 4.1

t	Exact solution	Computed solution	ERR	EFA	Eval t
0.1000	0.1102951969169624	0.1102951968849455	3.201692e-011	2.2551e-06	0.2436
0.2000	0.2419767996211093	0.2419767992452385	3.758708e-010	4.7763e-06	0.2454
0.3000	0.3951048486603785	0.3951048472221335	1.438245e-009	7.3083e-06	0.2472
0.4000	0.5678121662929389	0.5678121629380356	3.354903e-009	9.5635e-06	0.2490
0.5000	0.7560143934313761	0.7560143878578516	5.573525e-009	1.1301e-05	0.2508
0.6000	0.9535662164719235	0.9535662096167838	6.855140e-009	1.1301e-05	0.2526
0.7000	1.1529489669796242	1.1529489609377850	6.041839e-009	1.2408e-05	0.2545
0.8000	1.3463636553683762	1.3463636521999636	3.168413e-009	1.2940e-05	0.2565
0.9000	1.5269113132806256	1.5269113134142971	1.336715e-010	1.3100e-05	0.2584
1.0000	1.6894983915943840	1.6894983930867824	1.492398e-009	1.3245e-05	0.2602

Table 4.2. Showing the result for problem 4.2

t	Exact solution	Computed solution	ERR	EYH	Eval t
0.1000	0.0996679946249558	0.0996679946249443	1.149081e-014	4.1401e-07	0.1259
0.2000	0.1973753202249040	0.1973753202248368	6.716849e-014	6.0186e-07	0.1277
0.3000	0.2913126124515909	0.2913126124514075	1.833533e-013	7.3747e-07	0.1294
0.4000	0.3799489622552250	0.3799489622548863	3.386180e-013	1.7322e-07	0.1311
0.5000	0.4621171572600099	0.4621171572595237	4.861112e-013	6.8524e-07	0.1328
0.6000	0.5370495669980354	0.5370495669974555	5.798695e-013	7.9810e-07	0.1453
0.7000	0.6043677771171637	0.6043677771165689	5.948575e-013	9.2621e-07	0.1470
0.8000	0.6640367702678491	0.6640367702673163	5.327960e-013	2.8318e-07	0.1487
0.9000	0.7162978701990247	0.7162978701986086	4.161116e-013	6.6469e-07	0.1504
1.0000	0.7615941559557652	0.7615941559554906	2.745582e-013	7.2660e-07	0.1521

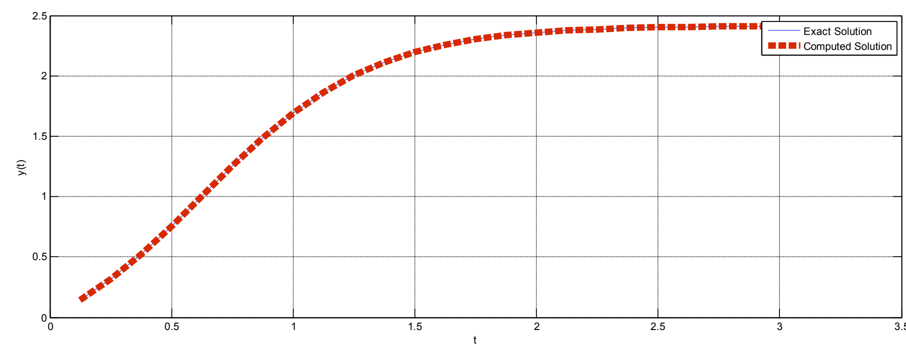

Fig. 2. Graphical results for problem 4.1

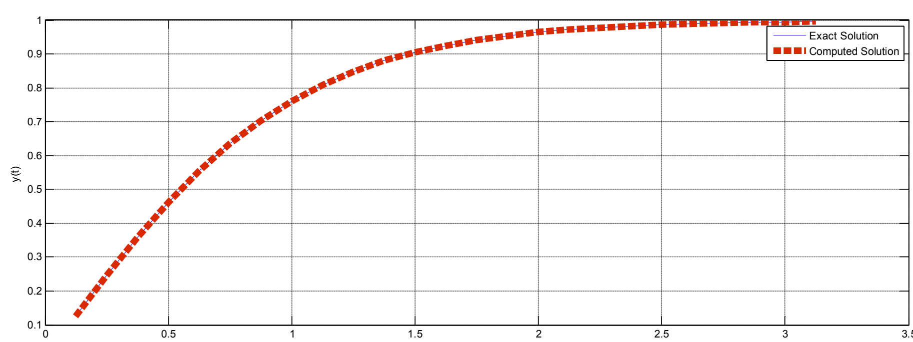

Fig. 3. Graphical results for problem 4.2

Problem 4.3:

Consider the Riccati differential equation,

$$y'(t) = -\frac{1}{1+t} + y(t) - y^2(t) \tag{27}$$

with the initial conditions,

$$y(0) = 1 \tag{28}$$

The exact solution is given by,

$$y(t) = \frac{1}{1+t} \tag{29}$$

Source: [19]

Problem 4.4:

Consider the Riccati differential equation,

$$y'(t) = 10 + 3y(t) - y^2(t) \tag{30}$$

whose initial conditions are,

$$y(0) = 0 \tag{31}$$

The exact solution is given by,

$$y(t) = -2 + \frac{14e^{7t}}{5+2e^{7t}} \tag{32}$$

Source: [25]

Problem 4.5:

Consider the Riccati differential equation,

$$y'(t) = y^2(t) - 1 \tag{33}$$

with the initial conditions,

$$y(0) = 0 \tag{34}$$

The exact solution is given by,

$$y(t) = -\tanh(t) \tag{35}$$

Source: [25]

Table 4.3. Showing the result for problem 4.3

t	Exact solution	Computed solution	ERR	EFA	Eval t
0.1000	0.9090909090909091	0.9090909090932011	2.292055e-012	3.8296e-07	0.0197
0.2000	0.8333333333333334	0.8333333333364473	3.113954e-012	3.8296e-07	0.0218
0.3000	0.7692307692307692	0.7692307692341456	3.376410e-012	5.7951e-07	0.0236
0.4000	0.7142857142857142	0.7142857142891383	3.424150e-012	6.8133e-07	0.0256
0.5000	0.6666666666666666	0.6666666666700610	3.394396e-012	7.3394e-07	0.0274
0.6000	0.6250000000000000	0.6250000000033436	3.343548e-012	7.6091e-07	0.0293
0.7000	0.5882352941176470	0.5882352941209419	3.294920e-012	7.7483e-07	0.0312
0.8000	0.5555555555555555	0.5555555555588129	3.257394e-012	7.8257e-07	0.0330
0.9000	0.5263157894736841	0.5263157894769185	3.234413e-012	7.8799e-07	0.0349
1.0000	0.4999999999999999	0.5000000000032264	3.226530e-012	7.9326e-07	0.0368

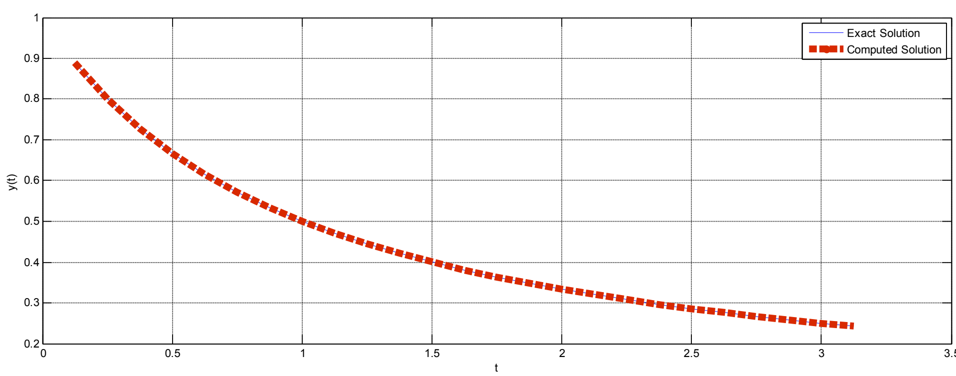

Fig. 4. Graphical results for problem 4.3

Table 4.4. Showing the result for problem 4.4

t	Exact solution	Computed solution	ERR	ENB	Eval t
0.1000	1.1229599550199856	1.1229599521930569	2.826929e-009	1.5×10^{-6}	0.0321
0.2000	2.3303636672393440	2.3303636731387738	5.899430e-009	3.2×10^{-6}	0.0493
0.3000	3.3592985913921902	3.3592986597014036	6.830921e-008	8.0×10^{-7}	0.0667
0.4000	4.0762561998939519	4.0762563498062434	1.499123e-007	3.2×10^{-6}	0.1056
0.5000	4.5086402379423145	4.5086404218874883	1.839452e-007	3.7×10^{-6}	0.1229
0.6000	4.7470598637518648	4.7470600293402532	1.655884e-007	9.7×10^{-7}	0.1419
0.7000	4.8720664654895440	4.8720665901929827	1.247034e-007	1.0×10^{-6}	0.1594
0.8000	4.9358801511182619	4.9358802354308153	8.431255e-008	8.5×10^{-7}	0.1766
0.9000	4.9680115179081801	4.9680115711478425	5.323966e-008	2.1×10^{-7}	0.1939
1.0000	4.9840783622386367	4.9840783943645039	3.212587e-008	1.4×10^{-6}	0.2985

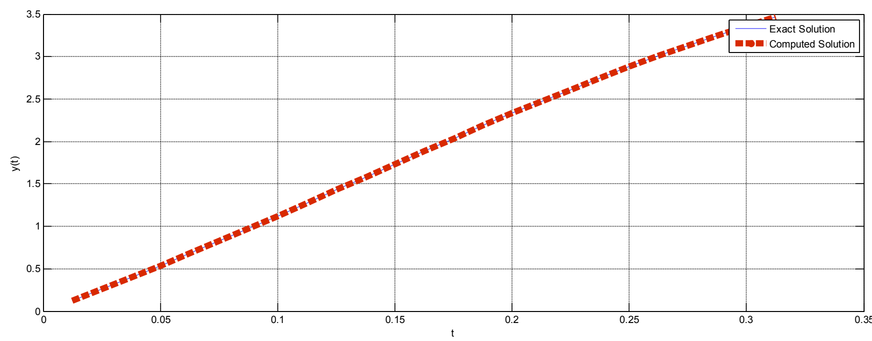

Fig. 5. Graphical results for problem 4.4

Table 4.5. Showing the result for problem 4.5

t	Exact solution	Computed solution	ERR	ENB	Eval t
0.1000	-0.0996679946249558	-0.0996679946249443	1.147693e-014	1.8×10^{-7}	0.2514
0.2000	-0.1973753202249040	-0.1973753202248368	6.714074e-014	1.2×10^{-6}	0.2530
0.3000	-0.2913126124515909	-0.2913126124514075	1.834088e-013	2.7×10^{-6}	0.2547
0.4000	-0.3799489622552249	-0.3799489622548863	3.385625e-013	3.5×10^{-6}	0.2564
0.5000	-0.4621171572600099	-0.4621171572595237	4.861112e-013	2.9×10^{-6}	0.2594
0.6000	-0.5370495669980354	-0.5370495669974555	5.798695e-013	1.6×10^{-6}	0.2610
0.7000	-0.6043677771171636	-0.6043677771165689	5.947465e-013	8.7×10^{-7}	0.2627
0.8000	-0.6640367702678492	-0.6640367702673163	5.329071e-013	9.2×10^{-7}	0.2645
0.9000	-0.7162978701990246	-0.7162978701986086	4.160006e-013	1.1×10^{-6}	0.2661
1.0000	-0.7615941559557651	-0.7615941559554906	2.744471e-013	1.8×10^{-7}	0.2678

5. DISCUSSION OF RESULTS

From the results above, it is obvious that the computational method derived is efficient in handling RDE and other first order differential equations of the form (1). The stability region obtained also shows that the method can effectively handle even stiff equations since it is L-stable. The evaluation time per seconds obtained were very small, showing that the method derived generates results faster. The analysis presented also show that the method is convergent, consistent and zero-stable.

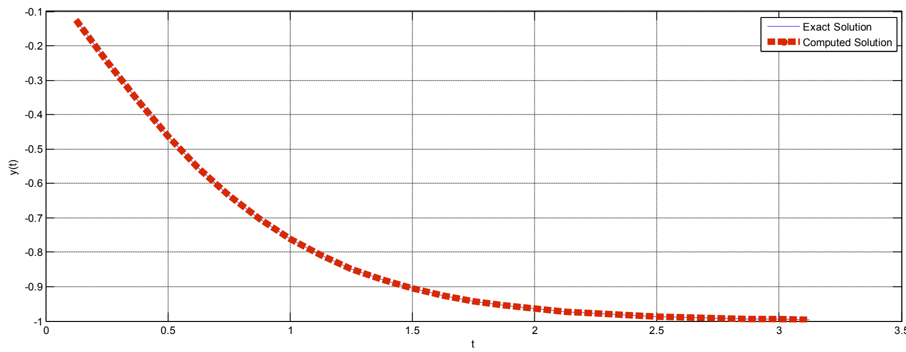

Fig. 6. Graphical results for problem 4.5

6. CONCLUSION

The method developed in this research has been shown to be efficient in solving RDEs of the form (1). Thus, the computational method is an alternative approach for solving RDEs.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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