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High Order Diminution of LTI System Using Stability Equation Method

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Authors' contributions

This work was carried out in collaboration between both authors. Author DKS designed the study, performed the statistical analysis, wrote the protocol, and wrote the first draft of the manuscript and managed literature searches. Authors DKS and GA both have managed the analyses of the study and literature searches. Both authors read and approved the final manuscript.

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Abstract

In this paper, a high-order transfer function is reduced by the stability equation method. The reduced model is based on the pole zero patterns. In this paper, the stability equation method is compared to other methods and proved to be better in comparison to other methods in literature. The analysis is based on the settling time, rise time and overshoot observations of the reduced and original system.

Keywords: Reduced Order Model (ROM) techniques; stability equation method; stability array method and mixed method.

1 Introduction

High-order system (HOS) is always costly and tedious. Model order reduction (MOR) is used for simplification of complicated problems in HOS. The selection of the model reduction technique is

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based on stability equation method as presented in [1]. The procedure of simplification is based on using mathematical approaches. Many methods to solve HOS for reduced order model (ROM) of the system are available in literature [2, 3]. The differentiation method for MOR of the systems has been proposed in [4]. It follows the simple steps of differentiation of the numerator and denominator polynomials but suffers with steady state error in the response of MOR as compared to original system [5, 6]. The methods in literature are not following the stability criterion except the stability equation method [7]. However, mixed method approach is prevalent to consider the stability effect in literature where the numerator and denominator are solved by using different/distinct method as in [8, 9, 10].

Recently, Desai and Prasad, have proposed the implementation of order reduction on TMS320C54X processor using genetic algorithm [11]. It also included fourth order example and reduced to 2nd order model. Pati at el, have proposed work on sub-optimal control using model order reduction [12]. Lodhwal and Jha, have proposed the performance comparison of different type of reduced order modeling methods [13, 14]. Model order reduction using stability equation method [15] and the continued fraction method [16] have been considered for deduction of MOR systems. Sambariya and Prasad, have proposed stable reduced model of a Single Machine Infinite Bus Power System (SMIB) with Power System Stabilizer, which is a seventh order system and being reduced to 2nd order ROM [17, 18, 19, 20].

MOR is used in industrial applications. In industrial application there are some similar measurement methods which minimize measurement error by transfer function method considering a different order transfer function of the system as presented in [21, 22, 23]. The selection of reduction technique is based on the closeness of the reduced order models response to higher order model response [24]. In literature, many authors have considered different methods to solve these problems. The Routh approximation method have been used in [25]. Alsmadi et al have presented the application of Sylvester based model order reduction for a multi-input multi-output (MIMO) power system [26]. It is based on the preservation of stability by retaining dominant poles and minimization of steady-state error. The application of PSO is examined in [25] by minimizing integral of square error of the response of original and reduced system. The genetic algorithm (GA) have been considered to determine the free coefficients of numerator and denominator of discrete transfer function in [27]. Soloklo, have presented the application of harmony search algorithm with multi-objective function for determining the lower order model of HO systems [28]. The application of Hermite polynomials with GA is presented for ROM of large systems in [29]. The stable reduced order modeling for linear time invariant systems is presented in [14, 19]. The application of cuckoo search is considered for deriving reduced order system of HOSs [30].

In this paper the application of stability equation method is presented for driving reduced order model of the higher order linear time invariant systems. The rest of paper is organized in four sections. The statement of problem along with the review on stability equation method is presented in section 2. The stability equation method is used for deriving the reduced order model and comparative study have been presented in section 3. The manuscript is concluded in section 4, followed by references.

2 Methodology for ROM

Consider a high order transfer function of a system represented as in Eqn. 2.1.

$$G(s) = \frac{\sum_{i=0}^{n-1} b_i s^i}{\sum_{i=0}^{n} a_i s^i} \tag{2.1}$$

where, the $G(s)$ represents a high order system with the order of n. The purpose of manuscript is to reduce the order of such high order system to r. The reduced order model may be represented as in Eqn. 2.2.

$$R(s) = \frac{\sum_{j=0}^{r-1} d_j s^j}{\sum_{j=0}^{r} c_j s^j} \tag{2.2}$$

where, a_i, b_i, c_j and d_j are the scalar constants of original high order system and the reduced order system. The objective is to find a reduced r^{th} order system model $R(s)$ such that it retains the important properties of $G(s)$ for the same types of inputs.

2.1 Steps for stability equation method

In this technique, the transfer function of reduced orders are obtained directly from the pole zero patterns of the stability equations of the original transfer function of system [17, 16]. Thus order of the stability equations of transfer function can be reduced [30]. Assuming, a high order transfer function the system is as shown in Eqn. 2.3.

$$G(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \ldots . b_1 s + b_0 + b_o}{a_n s^n + a_{n-1} s^{n-1} + \ldots . a_1 s + a_0} \tag{2.3}$$

$$G(s) = \frac{N(s)}{D(s)} \tag{2.4}$$

The Eqn. 2.4 is the symbolic representation of Eqn. 2.3. The $N(s)$ and $D(s)$ are the numerator and the denominator of $G(s)$, respectively. The order of $D(s)$ is n and the order of $N(s)$ is m such that the $n > m$.

2.1.1 Separation of even, odd parts and reduction

The numerator and the denominator of Eqn. 2.4 are separated in even and odd parts. If the even and odd parts are sub-scripted by e and o, respectively. The even and odd polynomials of numerator may be represented by $N_e(s)$ and $N_o(s)$, respectively. Similarly, the even and odd polynomials of denominator may be represented by $D_e(s)$ and $D_o(s)$, respectively. Therefore, the system in Eqn. 2.4 may be represented as following:

$$G(s) = \frac{N_e(s) + N_o(s)}{D_e(s) + D_o(s)} \tag{2.5}$$

$$G(s) = \frac{\sum_{i=0,2,4}^{m} b_i s^i + \sum_{i=1,3,5}^{m} b_i s^i}{\sum_{i=0,2,4}^{n} a_i s^i + \sum_{i=1,3,5}^{n} a_i s^i} \tag{2.6}$$

$$\begin{aligned} N_e(s) &= \sum_{i=0,2,4}^{m} b_i s^i \\ N_o(s) &= \sum_{i=1,3,5}^{m} b_i s_i \\ D_e(s) &= \sum_{i=0,2,4}^{n} a_i s_i \\ D_o(s) &= \sum_{i=1,3,5}^{n} a_i s_i \end{aligned} \tag{2.7}$$

The roots of $N_e(s)$ and $D_e(s)$ are called zeros $z_i(s)$ and that of $N_o(s)$ and $D_o(s)$ are called poles $p_i(s)$. In this method, a polynomial is reduced by successively discarding the less significant factors. Let us illustrate the method by reducing the denominator, the numerator is reduced similarly and the ratio of reduced numerator and denominator gives the reduced order model. The denominator is separated as following:

$$D(s) = D_e(s) + D_o(s) \tag{2.8}$$

where,

$$\begin{aligned} D_e(s) &= a_0 + a_2 s^2 + a_4 s^4 + \cdots \\ D_o(s) &= a_1 + a_3 s^3 + a_5 s^5 + \cdots \end{aligned} \tag{2.9}$$

The Eqn. 2.9 may be written as the following:

$$\begin{aligned} D_e(s) &= a_0 \prod_{i=1}^{k_1} \left(1 + \frac{s^2}{z_i^2}\right) \\ D_o(s) &= a_1 s \prod_{i=1}^{k_2} \left(1 + \frac{s^2}{p_i^2}\right) \end{aligned} \tag{2.10}$$

where k_1 and k_2 are integer parts of $n/2$ and $(n-2)/2$ respectively and $z12 < p12 < z22 < p22...$ by discarding the factors with larger magnitude of z_i and p_i, the reduced stability equations of desired order r become

$$\begin{aligned} D_{er}(s) &= a_o \prod_{i=1}^{r_1} \left(1 + \frac{s^2}{z_i^2}\right) \\ D_{or}(s) &= a_1 \prod_{i=1}^{r_2} \left(1 + \frac{s^2}{p_i^2}\right) \end{aligned} \tag{2.11}$$

where r_1 and r_2 are the integer parts of $r/2$ and $(r-1)/2$, respectively. The reduced denominator of the system is given as in Eqn. 2.12.

$$D_r(s) = D_{er}(s) + D_{or}(s) \tag{2.12}$$

The numerator of the reduced system may be represented as in following Eqn. 2.13.

$$N_r(s) = N_{er}(s) + N_{or}(s) \tag{2.13}$$

Similarly, the complete model of the r^{th} order reduced model may be represented as in Eqn. 2.14.

$$R(s) = \frac{N_{er}(s) + N_{or}(s)}{D_{er}(s) + D_{or}(s)} \tag{2.14}$$

It may be noted that poles and zeros with smaller magnitude are more dominant than those poles or zeros with larger magnitudes. The poles or zeros with larger magnitudes are discarded in this technique. Thus the reduced order models preserve the dominant performance of the original system.

3 Comparative Results and Discussion

3.1 System - 1

Considering a 8^{th} order system from literature survey as shown in Eqn. 3.1 [8].

$$G(s) = \frac{18s^7 + 514s^6 + 5928s^5 + 36380s^4 + 122664s^3 + 222088s^2 + 185760s + 40320}{s^8 + 36s^7 + 546s^6 + 4536s^5 + 22449s^4 + 67284s^3 + 118124s^2 + 185760s + 40320} \tag{3.1}$$

3.1.1 ROM by methods in literature

The 2^{nd} order ROM in literature using Ant Bee Colony Optimization (ABC) method as in [24].

$$G_2(s) = \frac{1.99s + 0.4318}{s^2 + 1.174s + 0.4318} \tag{3.2}$$

The 2^{nd} order ROM in literature using par Particle Swarm Optimization (PSO) method as in [31].

$$G_2(s) = \frac{88.0369s + 26.4768}{4.0214s^2 + 28.5882s + 2.6476} \tag{3.3}$$

3.1.2 ROM by proposed method

Reduction of numerator:

The 7^{th} order polynomial of the numerator of the original system is shown in the following Eqn. 3.4.

$$N_7(s) = 18s^7 + 514s^6 + 5982s^5 + 36380s^4 + 122664s^3 + 222088s^2 + 185760s + 40320 \tag{3.4}$$

Step-1: Reduction of numerator from 7^{th} order to 5^{th} order.

The arrangement of the odd and even terms is shown in the following Eqn. 3.5.

$$N_7(s) = \underbrace{18s^7 + 5982s^5 + 122664s^3 + 185760s}_{odd\,terms:N_o(s)} + \underbrace{514s^6 + 36380s^4 + 222088s^2 + 40320}_{even\,terms:N_e(s)} \tag{3.5}$$

$$\begin{aligned} N_o(s) &= 5982s^5\left(1 + \underbrace{\frac{18}{5982}}_{=0.003009} s^2\right) + 185760s\left(1 + \underbrace{\frac{122664}{185760}}_{=0.66033} s^2\right) \\ &= 5982s^5 + 122664s^3 + 185760s \end{aligned} \tag{3.6}$$

$$\begin{aligned} N_e(s) &= 36380s^4\left(1 + \underbrace{\frac{514}{36380}}_{=0.014128} s^2\right) + 40320\left(1 + \underbrace{\frac{222088}{40320}}_{=5.5081} s^2\right) \\ &= 36380s^4 + 222088s^2 + 40320 \end{aligned} \tag{3.7}$$

Combining above Eqns. 3.6-3.7 to get complete 5^{th} order system as following:

$$N_5(s) = 5982s^5 + 122664s^3 + 185760s + 36380s^4 + 222088s^2 + 40320 \tag{3.8}$$

Step-2: Reduction of 5^{th} order numerator to 3^{rd} order.

$$N_5(s) = \underbrace{5982s^5 + 122664s^3 + 185760s}_{odd} + \underbrace{36380s^4 + 222088s^2 + 40320}_{even} \tag{3.9}$$

$$\begin{aligned} N_o(s) &= 122664s^3\left(1+\underbrace{\frac{5982}{122664}}_{=0.04876}s^2\right)+185760s = 122664s^3+185760s \\ N_e(s) &= 222088s^2\left(1+\underbrace{\frac{36380}{222088}}_{=0.16380}s^2\right)+40320 = 222088s^2+40320 \end{aligned} \tag{3.10}$$

$$N_3(s) = 122664s^3+185760s+222088s^2+40320 \tag{3.11}$$

Step-3: Reduction of 3^{rd} order reduced numerator to 1^{st} order.

$$N_3(s) = \underbrace{122664s^3+185760s}_{odd}+\underbrace{222088s^2+40320}_{even} \tag{3.12}$$

$$\begin{aligned} &N_o(s) = 122664s^3+185760s = 185760s\left(1+\tfrac{122664}{185760}s^2\right) \\ &N_e(s) = 222088s^2+40320 = 40320\left(1+\tfrac{222088}{40320}s^2\right) \\ &N_2(s) = 185760s+40320 \end{aligned} \tag{3.13}$$

Reduction of denominator:

Step-1: Reduction of 8^{th} order denominator to 6^{th} order.

The arrangement of the odd and even terms is shown in the following Eqn. 3.14.

$$\begin{aligned} D_8(s) &= s^8+36s^7+546s^6+4536s^5+22449s^4+67284s^3+118124s^2+109584s+40320 \\ &= \underbrace{s^8+546s^6+22449s^4+118124s^2+40320}_{even}+\underbrace{36s^7+4536s^5+67284s^3+109584s}_{odd} \end{aligned} \tag{3.14}$$

$$\begin{aligned} D_e(s) &= s^8+546s^6+22449s^4+118124s^2+40320 \\ D_o(s) &= 36s^7+4536s^5+67284s^3+109584s \end{aligned} \tag{3.15}$$

$$\begin{aligned} D_e(s) &= s^8+546s^6+22449s^4+118124s^2+40320 \\ &= 546s^6\left(1+\underbrace{\frac{1}{546}}_{=0.00183}s^2\right)+118124s^2\left(1+\underbrace{\frac{22449}{118124}}_{0.19004}s^2\right)+40320 \\ &= 546s^6+22449s^4+118124s^2+40320 \end{aligned} \tag{3.16}$$

$$\begin{aligned} D_o(s) &= 36s^7+4536s^5+67284s^3+109584s \\ &= 4536s^5\left(1+\underbrace{\frac{36}{4536}}_{=0.00793}s^2\right)+109584s\left(1+\underbrace{\frac{67284}{109584}}_{=0.6139947}s^2\right) \\ &= 4536s^5+67284s^3+109584s \end{aligned} \tag{3.17}$$

$$D_6(s) = 546s^6+22449s^4+118124s^2+40320+4536s^5+67284s^3+109584s \tag{3.18}$$

Step-2: Reduction of 6^{th} order denominator to 4^{th} order.

$$D_6(s) = \underbrace{546s^6+22449s^4+118124s^2+40320}_{even}+\underbrace{4536s^5+67284s^3+109584s}_{odd} \tag{3.19}$$

$$\begin{aligned} D_e(s) &= 546s^6 + 22449s^4 + 118124s^2 + 40320 \\ &= 22449s^4 \left(1 + \underbrace{\frac{546}{22449}}_{=0.02432} s^2\right) + 40320 \left(1 + \underbrace{\frac{118124}{40320}}_{=2.9296627} s^2\right) \\ &= 22449s^4 + 118124s^2 + 40320 \end{aligned} \tag{3.20}$$

$$\begin{aligned} D_o(s) &= 4536s^5 + 67284s^3 + 109584s \\ &= 67284s^3 \left(1 + \underbrace{\frac{4536}{67284}}_{=0.067415} s^2\right) + 109584s \\ &= 67284s^3 + 109584s \end{aligned} \tag{3.21}$$

$$D_4(s) = 22449s^4 + 118124s^2 + 40320 + 67284s^3 + 109584s \tag{3.22}$$

Step-3: Reduction of 4^{th} order reduced denominator to 2^{nd} order.

$$D_4(s) = \underbrace{22449s^4 + 118124s^2 + 40320}_{even} + \underbrace{67284s^3 + 109584s}_{odd} \tag{3.23}$$

$$\begin{aligned} D_e(s) &= 22449s^4 + 118124s^2 + 40320 \\ &= 118124s^2 \left(1 + \underbrace{\frac{22449}{118124}}_{=0.19004605} s^2\right) + 40320 \\ &= 118124s^2 + 40320 \end{aligned} \tag{3.24}$$

$$\begin{aligned} D_o(s) &= 67284s^3 + 109584s \\ &= 109584s \left(1 + \underbrace{\frac{67284}{109584}}_{=0.6139947} s^2\right) \\ &= 109584s \end{aligned} \tag{3.25}$$

$$D_2(s) = 118124s^2 + 109584s + 40320 \tag{3.26}$$

The 2^{nd} reduced order model is given by using the Eqn. 3.13 and Eqn. 3.26

$$R_2(s) = \frac{N_2(s)}{D_2(s)} = \frac{185760s + 40320}{118124s^2 + 109584s + 40320} \tag{3.27}$$

3.1.3 Step response comparison

The systems, original, reduced by methods in literature and by proposed method are subjected to step response and compared as in Fig. 1. The simulation study with system is carried out in MATLAB 11b on computer configuration as Intel(R) Core(TM)2 Duo CPU T6400 @ 2 GhZ. It can be seen that the peak overshoot of the response for original system is very high as compared to proposed reduced model. The step-response of the proposed reduced order model is better as compared to other reduced models. The step-response information of the reduced order models along with original system is enlisted in Table 1. The step-response parameters such as rise time, settling time, peak and peak overshoot are comparable to ABC method [24] and are much improved as compared to PSO method [31]. The ABC and PSO are the soft computing methods while the proposed method is classical even of that the method provides better results.

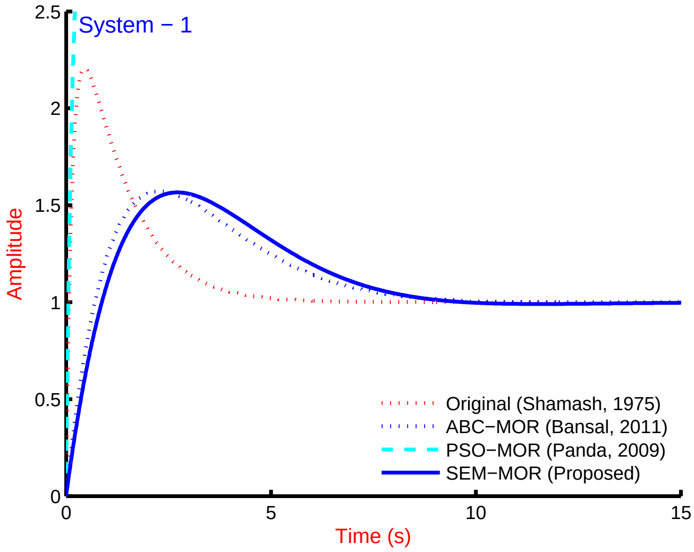

Fig. 1. Step response of original system as in Eqn. 3.1, ROM as in Eqns. 3.2 - 3.3 and proposed ROM using Stability equation method as in Eqn. 3.27

Table 1. Step response information of original system, ROM systems in literature and proposed ROM of system as system - 1

System	Rise Time (s)	Settling Time (s)	Peak	Peak Time (s)
Original (Eqn. 3.1)	0.0569	4.8201	2.2305	0.4493
ABC-ROM (Eqn. 3.2)	0.5515	8.7371	1.5717	2.3235
PSO-ROM (Eqn. 3.3)	20.6376	37.8414	9.9937	74.1211
SE (Prop.) (Eqn. 3.27)	0.6862	8.7170	1.5662	2.7142

3.2 System - 2

Consider a 4^{th} order example from literature survey as shown in Eqn. 3.28 [32].

$$G_4(s) = \frac{s^3 + 7s^2 + 24s + 24}{s^4 + 10s^3 + 35s^2 + 50s + 24} \tag{3.28}$$

3.2.1 ROM by methods in literature

The system in Eqn. 3.28 is reduced to the 2^{nd} order using genetic algorithm and represented as following in Eqn. 3.29 [11].

$$G_2(s) = \frac{0.7645s + 1.689}{s^2 + 2.591s + 1.689} \tag{3.29}$$

It is again reduced to the 2^{nd} order using sub-optimal concept in [12] and represented in Eqn. 3.30.

$$G_2(s) = \frac{s + 24.0096}{s^2 + 27.0096s + 24.0096} \tag{3.30}$$

3.2.2 ROM by proposed method

Reduction of numerator:
The numerator of the system - 2 is represented as in Eqn. 3.31 with its even and odd terms.

$$\begin{aligned} N(s) &= s^3 + 7s^2 + 24s + 24 \\ &= \underbrace{s^3 + 24s}_{odd} + \underbrace{7s^2 + 24}_{even} \end{aligned} \tag{3.31}$$

Step-1: Reduction of 3^{rd} order numerator to its 1^{st} order polynomial.

$$\begin{aligned} N(s) &= s^3 + 24s + 7s^2 + 24 \\ &= 24s\left(1 + \underbrace{\frac{1}{24}}_{=0.04167} s^2\right) + 24\left(1 + \underbrace{\frac{7}{24}}_{=0.29167} s^2\right) \\ &= 24s + 24 \end{aligned} \tag{3.32}$$

Reduction of denominator:

The denominator of the system - 2 is represented as in Eqn. 3.33 with its even and odd terms of powers of s operator.

$$\begin{aligned} D(s) &= s^4 + 10s^3 + 35s^2 + 50s + 24 \\ &= \underbrace{s^4 + 35s^2 + 24}_{even} + \underbrace{10s^3 + 50s}_{odd} \end{aligned} \tag{3.33}$$

Step-1: Reduction of 4^{th} order denominator to its 2^{nd} order polynomial using application of SE method.

$$\begin{aligned} D(s) &= 35s^2\left(1 + \underbrace{\frac{1}{35}}_{=0.0285} s^2\right) + 50s\left(1 + \underbrace{\frac{10}{50}}_{=0.2} s^2\right) + 24 \\ &= 35s^2 + 50s + 24 \end{aligned} \tag{3.34}$$

The 2^{nd} order reduced model by considering Eqn. 3.32 and Eqn. 3.2.2 is given as following in Eqn. 3.35.

$$R_2(s) = \frac{N_2(s)}{D_2(s)} = \frac{24s + 24}{35s^2 + 50s + 24} \tag{3.35}$$

3.2.3 Step response comparison

The response of 4^{th} order original system in [32] is compared to the response of 2^{nd} order reduced models presented in literature and derived by proposed SE method. The step response of these systems is shown in Fig. 2. It can be seen that the response of ROM with proposed method is retaining the properties of original system as in other models with new soft-computing methods. The step response data are enlisted in Table 2. It can be easily observed that the all parameters are equally relevant as with the original system.

3.3 System - 3

Consider a 4^{th} order example from literature survey as shown in Eqn. 3.36 [16].

$$G_4(s) = \frac{28s^3 + 496s^2 + 1800s + 2400}{2s^4 + 36s^3 + 204s^2 + 360s + 240} \tag{3.36}$$

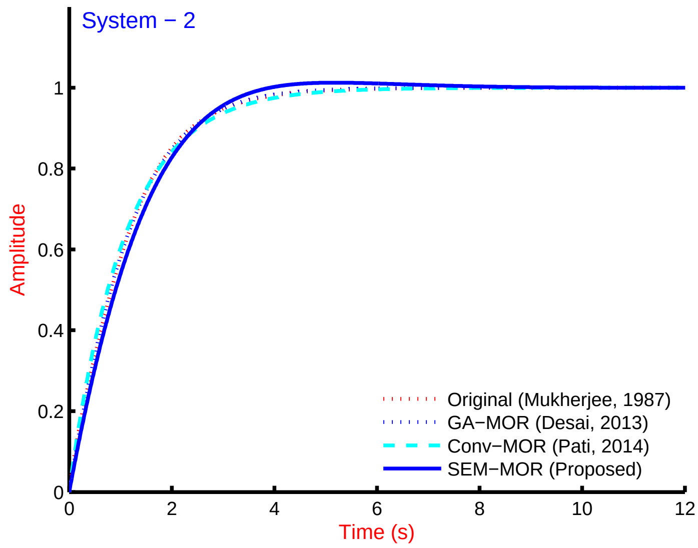

Fig. 2. Step response of original system as in Eqn. 3.28, ROM as in Eqns. 3.29 - 3.30 and proposed ROM using Stability equation method as in Eqn. 3.35

Table 2. Step response information of original system, ROM systems in literature and proposed ROM of system as System - 2

System	Rise Time (s)	Settling Time (s)	Peak	Peak Time (s)
Original (Eqn. 3.28)	2.2602	3.9307	0.9991	6.9770
GA-ROM (Eqn. 3.29)	2.2616	3.8443	1.0000	9.6778
Conv-ROM (Eqn. 3.30)	2.3875	4.2478	0.9996	8.4934
SE (Prop.) (Eqn. 3.35)	2.3008	3.3764	1.0125	5.2144

3.3.1 ROM by methods in literature

The system in Eqn. 3.36 is reduced to the 2^{nd} order using genetic algorithm and represented as following in Eqn. 3.37 [13].

$$R_2(s) = \frac{14s + 410.256}{s^2 + 1.785s + 1.19} \tag{3.37}$$

It is again reduced to the 2^{nd} order using sub-optimal concept in [16] and represented in Eqn. 3.38.

$$R_2(s) = \frac{8.93s + 1.19}{s^2 + 1.785s + 1.19} \tag{3.38}$$

3.3.2 ROM by proposed method

Reduction of numerator:

The numerator of the system - 3 is represented as in Eqn. 3.36 with its even and odd terms of powers of s operator.

$$N(s) = 28s^3 + 496s^2 + 1800s + 2400 \\ = \underbrace{28s^3 + 1800s}_{odd} + \underbrace{496s^2 + 2400}_{even} \tag{3.39}$$

Step-1: Reduction of 3^{rd} order numerator to its 1^{st} order polynomial is shown in Eqn. 3.40.

$$N(s) = 1800s\left(1 + \underbrace{\frac{28}{1800}}_{0.0156} s^2\right) + 2400\left(1 + \underbrace{\frac{496}{2400}}_{0.20667} s^2\right) \\ = 1800s + 2400 \tag{3.40}$$

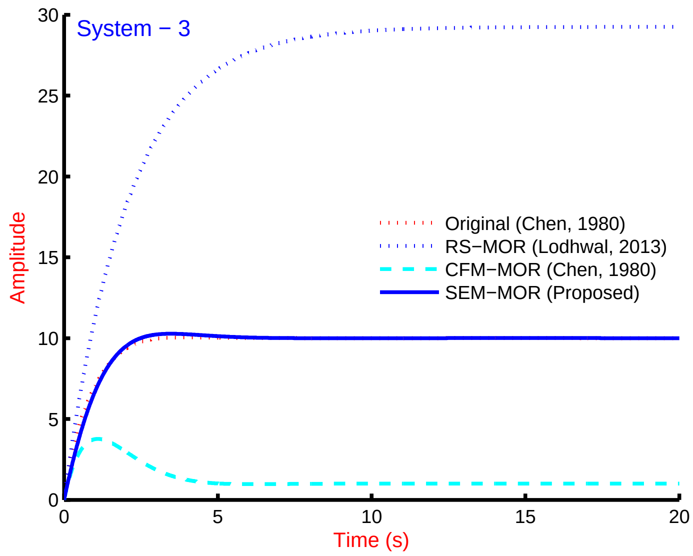

Fig. 3. Step response of original system as in Eqn. 3.36, ROM as in Eqns. 3.37 - 3.38 and proposed ROM using Stability equation method as in Eqn. 3.43

Table 3. Step response information of original system, ROM systems in literature and proposed ROM of system as System - 3

System	Rise Time (s)	Settling Time (s)	Peak	Peak Time (s)
Original (Eqn. 3.36)	1.6920	2.5514	10.0443	3.9460
RSA-ROM (Eqn. 3.37)	4.5602	8.1197	29.2387	16.2215
CFM-ROM (Eqn. 3.38)	0.0990	4.6801	3.7664	1.1168
SEM (Prop.) (Eqn. 3.43)	1.6150	4.3379	10.2711	3.4483

Reduction of denominator:
The denominator of the system - 3 is represented as in Eqn. 3.41 with its even and odd terms of powers of s operator.

$$\begin{aligned} D(s) &= 2s^4 + 36s^3 + 204s^2 + 360s + 240 \\ &= \underbrace{2s^4 + 204s^2 + 240}_{even} + \underbrace{36s^3 + 360s}_{odd} \end{aligned} \tag{3.41}$$

Step-1: Reduction of 4^{th} order denominator to its 2^{nd} order polynomial using application of SE method.

$$\begin{aligned} D(s) &= 204s^2\left(1 + \underbrace{\frac{2}{204}}_{=0.0098} s^2\right) + 360s\left(1 + \underbrace{\frac{36}{360}}_{0.10} s^2\right) + 240 \\ &= 204s^2 + 360s + 240 \end{aligned} \tag{3.42}$$

The 2^{nd} order model of original system in Eqn. 3.36 is obtained by combining the numerator and denominator derived in Eqn. 3.40 and Eqn. 3.42.

$$R_2 = \frac{1800s + 2400}{204s^2 + 360s + 240} \tag{3.43}$$

3.3.3 Step response comparison

The 4^{th} order system is reduced using stability equation method to the 2^{nd} order system. The comparison of original system, reduced order systems by methods in literature and that of with proposed method based reduced order model. The step response information is enlisted in Table 3 and the graphical response comparison is shown in Fig. 3. It can be seen that the ROM in [13] is not fulfilling the requirement of model order reduction. It is showing stable response but with very high steady-state error and settling time. The response that of with system in [16] is giving stable response and with steady-error. The response with original system and proposed reduced order model is reflecting almost with zero steady-state error. Therefore, it is an effective method in the arena of reduced order modeling of LTI systems.

4 Conclusion

The high order system is analyzed for it's reduced order model using proposed stability equation method. The step response of the reduced order model is compared with the systems in literature and with original systems of higher order. The step response information with proposed method is comparable to original higher order system in terms of rise time, settling time, peak and peak time. Further more, it preserves the stability in a very low order system. It is very easy method in the field of model order reduction as compared to other methods available in literature.

Competing Interests

The authors declare that no competing interests exist.

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